# Coherence of Semifilters

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*Theory of semifilters resembles*
*an iceberg whose visible part is*
*Theory of filters and ultrafilters*

### Motivation: Filters

A *filter* is a family $\mathcal{F}$ of subsets of $\omega$ such that:
(i) $\emptyset \notin \mathcal{F}$; (ii) $\mathcal{F} \ni A \subset B \subset \omega \Rightarrow B \in \mathcal{F}$; (iii) $A, B \in \mathcal{F} \Rightarrow A \cap B \in \mathcal{F}$.

A filter $\mathcal{F}$ on $\omega$ is *free* if $\cap\mathcal{F} = \emptyset$.

**Example:** The *Fréchet filter* $\mathfrak{Fr}$ of cofinite subsets of $\omega$.

$\mathfrak{Fr}$ is the smallest element of the set $\mathsf{FF}$ of free filters, ordered by inclusion relation.

This set has no largest element, but has $2^{\mathfrak{c}}$ maximal elements called *ultrafilters*.

### Motivation: Near Coherence of Filters

The Blass' principle NCF, the **N**ear **C**oherence of **F**ilters, asserts that any two free filters $\mathcal{F},\mathcal{U}$ on $\omega$ are *near coherent* in the sense that for some finite-to-one function $\varphi : \omega \to \omega$ the images $\varphi(\mathcal{F})$ and $\varphi(\mathcal{U})$ are linked which means that $\varphi(F) \cap \varphi(U) \neq \emptyset$ for all $F \in \mathcal{F}$ and $U \in \mathcal{U}$.

NCF has many applications beyond Set Theory...

The consistency of NCF was proved by A.Blass and S.Shelah in 1987 who constructed a model of ZFC satisfying three formally stronger principles than NCF:

**(Ultrafilter Monotomy)** For any ultrafilters $\mathcal{U}_0$ and $\mathcal{F}$ on $\omega$ there is a monotone surjection $\varphi : \omega \to \omega$ such that $\varphi(\mathcal{F}) = \varphi(\mathcal{U}_0)$.

**(Filter Dichotomy)** For any ultrafilter $\mathcal{U}_0$ and any filter $\mathcal{F}$ on $\omega$ there is a monotone surjection $\varphi : \omega \to \omega$ such that either $\varphi(\mathcal{F}_0) = \varphi(\mathcal{U}_0)$ or $\varphi(\mathcal{F}) = \mathfrak{Fr}$.

**(Semifilter Trichotomy)** For any ultrafilter $\mathcal{U}_0$ and any semifilter $\mathcal{F}$ on $\omega$ (= a family of infinite subsets of $\omega$ closed under taking almost supersets) there is a monotone surjection $\varphi : \omega \to \omega$ such that $\varphi(\mathcal{F})$ coincides with $\varphi(\mathcal{U}_0)$, $\mathfrak{Fr}$, or $\mathfrak{Fr}^\perp = [\omega]^\omega$.

It is known that $(\mathfrak{u} < \mathfrak{g}) \Rightarrow$ (S3) $\Rightarrow$ (F2) $\Rightarrow$ (U1) $\Leftrightarrow$ (NCF) $\Rightarrow (\mathfrak{u} < \mathfrak{d})$

### Introducing Semifilters

By a *semifilter* we understand any family $\mathcal{F}$ of infinite subsets of $\omega$ closed under taking almost supersets. So, $\mathcal{F}$ is a *semifilter* if: (i) $\emptyset \notin \mathcal{F}$ and (ii) $\mathcal{F} \ni A \subset^* B \;\Rightarrow\; B \in \mathcal{F}$.

Adding to those 2 conditions the third: (iii) $A, B \in \mathcal{F} \;\Rightarrow\; A \cap B \in \mathcal{F}$, we get the definition of a free filter.

Semifilters have some advantages comparing to filters. In particular, for any semifilters $\mathcal{F},\mathcal{U}$ the intersection $\mathcal{F} \cap \mathcal{U}$ and the union $\mathcal{F} \cup \mathcal{U}$ are semifilters. Besides these two operations there is an important operation of *transversal*: $\mathcal{F}^\perp = \{E \subset \omega : \forall F \in \mathcal{F}\; F \cap E \neq \emptyset\}$.

In partucular, the transversal semifilter $\mathfrak{Fr}^\perp$ to the Fréchet filter is the semifilter consisting of all infinite subsets of $\omega$. Also $\mathcal{F}^\perp = \mathcal{F}$ for any ultrafilter.

The operation of transversal has algeraic properties: (i) $(\mathcal{F}^\perp)^\perp = \mathcal{F}$, (ii) $(\mathcal{F} \cup \mathcal{U})^\perp = \mathcal{F}^\perp \cap \mathcal{U}^\perp$, (iii) $(\mathcal{F} \cap \mathcal{U})^\perp = \mathcal{F}^\perp \cup \mathcal{U}^\perp$.

The *support* of a semifilter $\mathcal{F}$ is the filter: $\mathrm{supp}(\mathcal{F}) = \{S \subset \omega : \forall F \in \mathcal{F}\; F \cap S \in \mathcal{F}\}$.

**Fact:** $\mathrm{supp}(\mathcal{F}) = \mathcal{F}$ iff $\mathcal{F}$ is a filter.

### The lattice $\mathsf{SF}$ of semifilters

By $\mathsf{SF}$ we denote the family of all semifilters on $\omega$. This family is a sublattice of the double power-set $\mathcal{P}(\mathcal{P}(\omega))$, considered as a complete lattice with respect to the operations of union and intersection.

The semifilters $\mathfrak{Fr}$ and $\mathfrak{Fr}^\perp$ are the smallest and the largest elements of this lattice and the operation $\perp: \mathsf{SF} \to \mathsf{SF}$ is an involutive anti-isomorphism of $\mathsf{SF}$.

**Desribing self-dual semifilters.** Let $\mathcal{F}$ be a semifilter.
- $\mathcal{F} \subset \mathcal{F}^\perp$ iff $\mathcal{F}$ is *linked* ($\forall A, B \in \mathcal{F}\;\; A \cap B \neq \emptyset$);
- $\mathcal{F}^\perp = \mathcal{F}$ iff $\mathcal{F}$ is *maximal linked*.

For any semifilter $\mathcal{F}$ there is a maximal linked semifilter $\mathcal{L}$ with $\mathcal{F} \cap \mathcal{F}^\perp \subset \mathcal{L} \subset \mathcal{F} \cup \mathcal{F}^\perp$.

## Finite-to-Finite Multifunctions

Recall that two ultrafilters $\mathcal{F},\mathcal{U}$ are near coherent if $\varphi(\mathcal{F}) \subset \varphi(\mathcal{U})$ for some finite-to-one function $\varphi : \omega \to \omega$. This is equivalent to saying that $\varphi^{-1} \circ \varphi(\mathcal{F}) \subset \mathcal{U}$, where $\varphi^{-1} \circ \varphi : x \mapsto \varphi^{-1} \circ \varphi(x)$ is an example of a finite-to-finite multifunction.

By a *multifunction* from a set $X$ to a set $Y$ we shall understand a subset $\Phi \subset X \times Y$ that can be thought as a multivalued function $\Phi : X \Rightarrow Y$ assigning to each point $x \in X$ the set $\Phi(x) = \{y \in \Phi : (x, y) \in \Phi\}$.

Such a multifunction $\Phi$ is *finite-to-finite* if for any $x \in X$ and $y \in Y$ the sets $\Phi(x)$ and $\Phi^{-1}(y)$ are finite and non-empty.

Multifunctions have some advantages comparing to usual functions because the family of multifunctions on $\omega$ is closed with respect to taking inverses and unions!

## Subcoherence relation $\Subset$ on $\mathsf{SF}$

A semifilter $\mathcal{F}$ is *subcoherent* to a semifilter $\mathcal{U}$ (denoted by $\mathcal{F} \Subset \mathcal{U}$) if $\Phi(\mathcal{F}) \subset \mathcal{U}$ for some finite-to-finite multifunction $\Phi : \omega \Rightarrow \omega$. If $\mathcal{F} \Subset \mathcal{U}$ and $\mathcal{U} \Subset \mathcal{F}$, then we say that $\mathcal{F}$ is *coherent* to $\mathcal{U}$ and write $\mathcal{F} \asymp \mathcal{U}$.

**Theorem** (Talagrand). For a semifilter $\mathcal{F}$ TFAE: (i) $\mathcal{F} \asymp \mathfrak{Fr}$; (ii) $\varphi(\mathcal{F}) = \mathfrak{Fr}$ for some monotone surjection $\varphi : \omega \to \omega$; (iii) $\mathcal{F}$ is a meager subset in the power-set $\mathcal{P}(\omega)$.

**Dual Theorem.** For a semifilter $\mathcal{F}$ TFAE: (i) $\mathcal{F} \asymp \mathfrak{Fr}^\perp$, (ii) $\varphi(\mathcal{F}) = \mathfrak{Fr}^\perp$ for some monotone surjection $\varphi : \omega \to \omega$, (iii) $\mathcal{F}$ is a comeager subset in $\mathcal{P}(\omega)$.

A semifilter $\mathcal{F}$ is *bi-Baire* if both $\mathcal{F}$ and $\mathcal{F}^\perp$ are Baire; equivalently, if $\mathfrak{Fr} \not\asymp \mathcal{F} \not\asymp \mathfrak{Fr}^\perp$. Each semifilter $\mathcal{F} \asymp \mathcal{F}^\perp$ is bi-Baire.

## The coherence lattice $[\mathsf{SF}]$

The subcoherence relation $\Subset$ on the semifilter lattice $\mathsf{SF}$ is reflexive and transitive, so the coherence relation $\asymp$ is an equivalence relation on $\mathsf{SF}$. This equivalence relation is a congruence on $\mathsf{SF}$ because $\Subset$ nicely agrees with the algebraic operations on $\mathsf{SF}$: If $\mathcal{F} \Subset \mathcal{F}', \mathcal{U} \Subset \mathcal{U}'$, then $(\mathcal{F}')^\perp \Subset \mathcal{F}$, $\mathcal{F} \cup \mathcal{U} \Subset \mathcal{F}' \cup \mathcal{U}'$ and $\mathcal{F} \cap \mathcal{U} \Subset \mathcal{F} \cap \mathcal{U}$.

This allows us to consider the quotient lattice $[\mathsf{SF}] = \mathsf{SF}/_\asymp$ whose elements are coherence classes $[\mathcal{F}] = \{\mathcal{U} \in \mathsf{SF} : \mathcal{U} \asymp \mathcal{F}\}$ of semifilters $\mathcal{F} \in \mathsf{SF}$. Besides two lattice operations $[\mathcal{F}] \cup [\mathcal{U}] = [\mathcal{F} \cup \mathcal{U}]$ and $[\mathcal{F}] \cap [\mathcal{U}] = [\mathcal{F} \cap \mathcal{U}]$ the lattice $[\mathsf{SF}]$ carries the operation of transversal $[\mathcal{F}]^\perp = [\mathcal{F}^\perp]$.

The lattice $[\mathsf{SF}]$ called the *coherence lattice* is an extremely interesting set-theoretic object: like a chameleon it changes its properties depending on additional set-theoretic axiom. For example, $[\mathsf{SF}]$ has only 3 elements under $(\mathfrak{u} < \mathfrak{g})$ and contains a copy of $\mathsf{SF}$ under $(\mathfrak{u} \geq \mathfrak{d})$.

## Cardinal Functions on $[\mathsf{SF}]$: General Theory

A cardinal function $\xi(-)$ on $\mathsf{SF}$ is called

- $\cup$-*homomorphism* if $\xi(\mathcal{F} \cup \mathcal{U}) = \max\{\xi(\mathcal{F}), \xi(\mathcal{U})\}$ for all semifilters $\mathcal{F},\mathcal{U}$;
- $\cap$-*homomorphism* if $\xi(\mathcal{F} \cap \mathcal{U}) = \min\{\xi(\mathcal{F}), \xi(\mathcal{U})\}$ for all semifilters $\mathcal{F},\mathcal{U}$;
- $\subset$-*monotone* if $\xi(\mathcal{F}) \leq \xi(\mathcal{U})$ for any semifilters $\mathcal{F} \subset \mathcal{U}$;
- $\Subset$-*monotone* if $\xi(\mathcal{F}) \leq \xi(\mathcal{U})$ for any semifilters $\mathcal{F} \Subset \mathcal{U}$;
- $\asymp$-*invariant* if $\xi(\mathcal{F}) = \xi(\mathcal{U})$ for any semifilters $\mathcal{F} \asymp \mathcal{U}$.

For a cardinal function $\xi(-)$ on $\mathsf{SF}$ there are (at least) 3 ways to produce an $\asymp$-invariant cardinal function:

- $\xi_{[\mathcal{F}]} = \min\{\xi(\mathcal{U}) : \mathcal{U} \in [\mathcal{F}]\}$, the *minimization*,
- $\xi^{[\mathcal{F}]} = \sup\{\xi(\mathcal{U}) : \mathcal{U} \in [\mathcal{F}]\}$, the *supremization*,
- $\hat{\xi}[\mathcal{F}] = \min\{\xi^{\mathsf{SF}}, \xi(\mathcal{U}) : \mathcal{U} \notin \mathcal{F}\}$, the *nonification* of $\xi(-)$.

The minimization $\xi_{[-]}$ and supremization $\xi^{[-]}$ are $\cup$-homomorphisms on $\mathsf{SF}$ if so is $\xi(-)$.

The nonification $\hat{\xi}[-]$ of any cardinal function $\xi(-)$ is a $\Subset$-monotone $\cap$-homomorphism on $\mathsf{SF}$.

**Trivial but Important Fact:** $\mathcal{F} \Subset \mathcal{U}$ if $\xi_{[\mathcal{F}]} < \hat{\xi}[\mathcal{U}]$.

## Critical values of cardinal functions

For a class of semifilters $\mathsf{F} \subset \mathsf{SF}$ let $\xi_{\mathsf{F}} = \min\{\xi(\mathcal{F}) : \mathcal{F} \in \mathsf{F}\}$ and $\xi^{\mathsf{F}} = \sup\{\xi(\mathcal{F}) : \mathcal{F} \in \mathsf{F}\}$ be the critical values of $\xi(-)$ on $\mathsf{F}$.

Among such classes $\mathsf{F}$ the most important is the class $\mathsf{ML}$ of all maximal linked semifilters. The critial values $\xi_{\mathsf{ML}}$ and $\hat{\xi}^{\mathsf{ML}}$ play a special role because of **Polarization Formulas:**

- If $\xi(-)$ is $\subset$-monotone, then $\min\{\xi_{[\mathcal{F}]}, \hat{\xi}[\mathcal{F}^\perp]\} \leq \xi_{\mathsf{ML}}$ and $\max\{\xi_{[\mathcal{F}]}, \hat{\xi}[\mathcal{F}^\perp]\} \geq \hat{\xi}^{\mathsf{ML}}$.
- If $\xi(-)$ is a $\cup$-homomorphism, then $\max\{\xi_{[\mathcal{F}]}, \xi_{[\mathcal{F}^\perp]}\} \geq \xi_{\mathsf{ML}}$ and $\min\{\hat{\xi}[\mathcal{F}], \hat{\xi}[\mathcal{F}^\perp]\} \leq \hat{\xi}^{\mathsf{ML}}$.

**Def.** A semifilter $\mathcal{F}$ is called $\xi$-*minimal* ($\hat{\xi}$-*maximal*) if $\max\{\xi_{[\mathcal{F}]}, \xi_{[\mathcal{F}^\perp]}\} \leq \xi_{\mathsf{ML}}$ $(\min\{\hat{\xi}[\mathcal{F}], \hat{\xi}[\mathcal{F}^\perp]\} \geq \hat{\xi}^{\mathsf{ML}})$.

### Two Fundamental Theorems

**Th. I.** If $\xi_{\mathsf{ML}} < \hat{\xi}^{\mathsf{ML}}$, then for a semifilter $\mathcal{F}$ the following conditions are equivalent: (1) $\mathcal{F}$ is $\xi$-minimal; (2) $\mathcal{F}$ is $\hat{\xi}$-maximal; (3) $\max\{\xi_{[\mathcal{F}]}, \xi_{[\mathcal{F}^\perp]}\} < \hat{\xi}^{\mathsf{ML}}$; (4) $\min\{\hat{\xi}[\mathcal{F}], \hat{\xi}[\mathcal{F}^\perp]\} > \xi_{\mathsf{ML}}$.
Moreover, all semifilters $\mathcal{F}$ with properties (1)–(4) are coherent.

**Th. II.** If $\xi_{\mathsf{ML}} < \widehat{\hat{\xi}}^{\mathsf{ML}}$, then $\mathsf{SF}$ contains at most two non-coherent maximal linked semifilters. More precisely, a semifilter $\mathcal{L}$ with $\hat{\xi}[\mathcal{L}] = \hat{\xi}[\mathcal{L}^\perp]$ is coherent to a unique $\xi$-minimal (resp. $\hat{\xi}$-maximal) maximal linked semifilter if $\hat{\xi}[\mathcal{L}] > \xi_{\mathsf{ML}}$ (resp. $\hat{\xi}[\mathcal{L}] < \widehat{\hat{\xi}}^{\mathsf{ML}}$).

### Cardinal characteristics of semifilters: four levels of complexity

The cardinal characteristics of semifilters appearing in practice fall into four complexity categories:

(1) Cardinal characteristics of semifilters determined by their inner structure (as a rule they are not $\asymp$-invariant);
(2) $\asymp$-Invariant cardinal characteristics obtained after minimizations or supremizations of cardinal characteristics of the first level;
(3) Cardinal characteristics obtained by nonifications of cardinal characteristics at 2d complexity level;
(3') Cardinal characteristics of some external objects determined by a semifilter, close by their properties to the cardinal characteristics of the third level;
(4) Cardinal characteristics obtained after nonification of the cardinal characteristics at 3d complexity level.

### The $\pi$-character of a semifilter

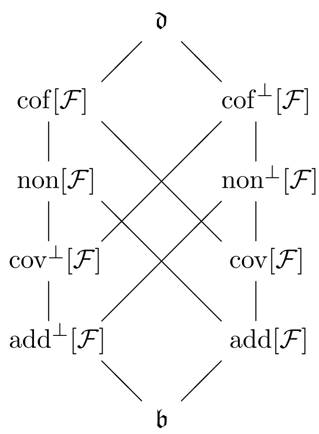
For a semifilter $\mathcal{F}$ let $\pi\chi(\mathcal{F}) = \min\{|\mathcal{B}| : \mathcal{B} \subset \mathfrak{Fr}^\perp\ \forall F \in \mathcal{F} \exists B \in \mathcal{B}\ B \subset^* F\}$ be the *$\pi$-character* of $\mathcal{F}$.
The $\pi$-character $\pi\chi(-)$ is a $\cup$-homomorphism on $\mathsf{SF}$ and so is its minimization $\pi\chi_{[-]}$.

**Critical values:** $\pi\chi_{\mathsf{ML}} = \mathfrak{r}$ (Balcar-Simon); If $(\mathfrak{r} < \mathfrak{d})$, then $\widehat{\pi\chi}^{\mathsf{ML}} = \mathfrak{d}$.
**Polarization Formulas for $\pi\chi(-)$:** $\max\{\pi\chi_{[\mathcal{F}]}, \pi\chi_{[\mathcal{F}^\perp]}\} \geq \mathfrak{r}$ and $\min\{\pi\chi_{[\mathcal{F}]}, \widehat{\pi\chi}[\mathcal{F}^\perp]\} \leq \mathfrak{r}$,
If $(\mathfrak{r} < \mathfrak{d})$, then • $\min\{\widehat{\pi\chi}[\mathcal{F}], \widehat{\pi\chi}[\mathcal{F}^\perp]\} \leq \mathfrak{d}$ and $\max\{\pi\chi_{[\mathcal{F}]}, \widehat{\pi\chi}[\mathcal{F}^\perp]\} \geq \mathfrak{d}$.

### "Ideal" cardinal characteristics

For a semifilter $\mathcal{F} \not\asymp \mathfrak{Fr}^\perp$ let
• $\mathrm{add}[\mathcal{F}] = \min\{|\mathsf{C}| : \mathsf{C} \subset [\mathcal{F}],\ \cup\mathsf{C} \notin [\mathcal{F}]\}$;
• $\mathrm{cov}[\mathcal{F}] = \min\{|\mathsf{C}| : \mathsf{C} \subset [\mathcal{F}],\ \cup\mathsf{C} = \mathfrak{Fr}^\perp\}$;
• $\mathrm{cof}[\mathcal{F}] = \min\{|\mathsf{C}| : \mathsf{C} \subset [\mathcal{F}]\ \forall \mathcal{F}' \in [\mathcal{F}]\ \exists \mathcal{C} \in \mathsf{C}\ \ \mathcal{F}' \subset \mathcal{C}\}$;
• $\mathrm{non}[\mathcal{F}] = \min\{|\mathcal{C}| : \mathcal{C} \subset \mathfrak{Fr}^\perp,\ \mathcal{C} \notin \mathcal{F}\}$.

These cardinal characteristics relate as follows:




Here $\xi^\perp[\mathcal{F}] = \xi[\mathcal{F}^\perp]$ for $\xi \in \{\mathrm{add}, \mathrm{cov}, \mathrm{cof}, \mathrm{non}\}$. Looking at this diagram we may complete the definition of $\mathrm{add}[\mathcal{F}]$, $\mathrm{cov}[\mathcal{F}]$, $\mathrm{non}[\mathcal{F}]$, $\mathrm{cof}[\mathcal{F}]$ letting $\mathrm{add}[\mathfrak{Fr}^\perp] = \mathrm{cov}[\mathfrak{Fr}^\perp] = \mathfrak{b}$ and $\mathrm{non}[\mathfrak{Fr}^\perp] = \mathrm{cof}[\mathfrak{Fr}^\perp] = \mathfrak{d}$.

**The nature of $\widehat{\pi\chi}[-]$**

**A crucial observation:** $\mathrm{non}[\mathcal{F}] = \widehat{\pi\chi}[\mathcal{F}]$ for any semifilter $\mathcal{F} \not\leq \mathfrak{Fr}^\perp$.

The Polarization Formulas for $\pi\chi_{[-]}$ improve to: $\min\{\pi\chi_{[\mathcal{F}]}, \mathrm{non}^\perp[\mathcal{F}]\} \leq \mathfrak{r}$ and $\max\{\pi\chi_{[\mathcal{F}]}, \mathrm{cov}[\mathcal{F}]\} \geq \mathfrak{d}$.

**The relation $\leq_\mathcal{F}$**

By $\omega^{\uparrow\omega}$ we denote the set of all non-decreasing unbounded functions from $\omega$ to $\omega$.

For two functions $x, y \in \omega^{\uparrow\omega}$ and a semifilter $\mathcal{F}$ we write $x \leq_\mathcal{F} y$ if $\{n \in \omega : x(n) \leq y(n)\} \in \mathcal{F}$. Let

- $\mathfrak{d}(\mathcal{F}) = \min\{|D| : D \subset \omega^{\uparrow\omega}\ \forall x \in \omega^{\uparrow\omega}\ \exists y \in D\ \ x \leq_\mathcal{F} y\}$;
- $\mathfrak{b}(\mathcal{F}) = \min\{|D| : D \subset \omega^{\uparrow\omega}\ \forall x \in \omega^{\uparrow\omega}\ \exists y \in D\ \ y \not\leq_\mathcal{F} x\}$;
- $\mathfrak{q}(\mathcal{F}) = \min\{|D| : D \subset \omega^{\uparrow\omega}\ \forall x \in \omega^{\uparrow\omega}\ \exists y \in D\ \ x \not\leq_\mathcal{F} y\}$.

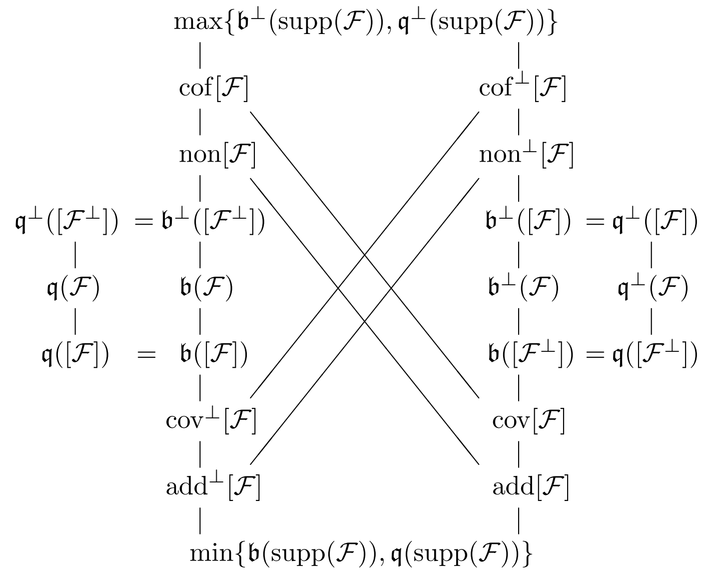
**Important observation:** $\mathfrak{d}(\mathcal{F}) = \mathfrak{b}(\mathcal{F}^\perp)$, so we can exclude $\mathfrak{d}(\mathcal{F})$ from consideration.

**Proposition.** The cardinal characteristics $\mathfrak{b}(-)$ and $\mathfrak{q}(-)$ are $\Subset$-monotone lattice homomorphisms on $\mathsf{SF}$.

These cardinal characteristics are particular cases of cardinal functions $\mathfrak{b}(\mathsf{F})$, $\mathfrak{q}(\mathsf{F})$ defined for a class $\mathsf{F} \subset \mathsf{SF}$ of semifilters as follows:

- $\mathfrak{b}^\perp(\mathsf{F}) = \min\{|D| : D \subset \omega^{\uparrow\omega}\quad \forall(\mathcal{F}, f) \in \mathsf{F} \times \omega^{\uparrow\omega}\ \exists g \in D$ with $f \leq_\mathcal{F} g\}$;
- $\mathfrak{q}^\perp(\mathsf{F}) = \min\{|D| : D \subset \omega^{\uparrow\omega}\quad \forall(\mathcal{F}, f) \in \mathsf{F} \times \omega^{\uparrow\omega}\ \exists g \in D$ with $g \leq_\mathcal{F} f\}$;
- $\mathfrak{b}(\mathsf{F}) = \min\{|\mathsf{P}| : \mathsf{P} \subset \mathsf{F} \times \omega^{\uparrow\omega}\quad \forall g \in \omega^{\uparrow\omega}\ \exists(\mathcal{F}, f) \in \mathsf{P}$ with $f \not\leq_\mathcal{F} g\}$;
- $\mathfrak{q}(\mathsf{F}) = \min\{|\mathsf{P}| : \mathsf{P} \subset \mathsf{F} \times \omega^{\uparrow\omega}\quad \forall g \in \omega^{\uparrow\omega}\ \exists(\mathcal{F}, f) \in \mathsf{P}$ with $g \not\leq_\mathcal{F} f\}$.

**Interplay between cardinal caharacteristics** of a semifilter $\mathcal{F}$ are described by the diagram:




**Corollary:**

1) If $\mathcal{F}$ is a filter, then $\mathrm{add}^\perp[\mathcal{F}] = \mathrm{cov}^\perp[\mathcal{F}] = \min\{\mathfrak{b}[\mathcal{F}], \mathfrak{q}[\mathcal{F}]\}$.

2) If $\mathcal{F}^\perp$ is a filter, then $\mathrm{cof}[\mathcal{F}] = \mathrm{non}[\mathcal{F}] = \max\{\mathfrak{b}[\mathcal{F}], \mathfrak{q}[\mathcal{F}]\}$.

3) If $\mathcal{F}$ is an ultrafilter, then $\mathrm{add}[\mathcal{F}] = \mathrm{cov}[\mathcal{F}] = \min\{\mathfrak{b}[\mathcal{F}], \mathfrak{q}[\mathcal{F}]\}$ and $\mathrm{cof}[\mathcal{F}] = \mathrm{non}[\mathcal{F}] = \max\{\mathfrak{b}[\mathcal{F}], \mathfrak{q}[\mathcal{F}]\}$.

4) If $\mathcal{F}$ is an ultrafilter, coherent to no $Q$-point, then all the cardinals from the diagram are equal.

**References**


1. T.Banakh, L.Zdomskyy. Coherence of Semifilters,
http://www.franko.lviv.ua/faculty/mechmat/Departments/Topology/booksite.html