On measures on convex Corson compacta

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Abstract

A topological space K is said to be *Corson compact* if K is homeomorphic to a compact subset of

$$\Sigma(\mathbb{R}^\kappa) = \{x \in \mathbb{R}^\kappa : \ |\{\alpha < \kappa : x(\alpha) \neq \emptyset\}| \leq \omega\},$$

for some κ. For a given space K, we denote by $P(K)$ the set of all probability Radon measures on K.

It is well–known that under MA(ω_1) every measure $\mu \in P(K)$, where K is Corson compact, has metrizable support. On the other hand, the results due to Haydon, Kunen and others show that under the continuum hypothesis (or some weaker axioms) there are nonseparable Corson compact spaces, supporting Radon measures.

If L is a compact and **convex** subset of the positive part $\Sigma(\mathbb{R}^\kappa_+)$ of $\Sigma(\mathbb{R}^\kappa)$, then an easy Krein–Millman type argument shows (in ZFC) that every measure $\mu \in P(K)$ lives on some metrizable subspace of K.

Convexity, however, does not help in the general case:

Theorem (CH). *There are a compact convex subset L of $\Sigma(\mathbb{R}^{\omega_1})$, and a measure $\mu \in P(L)$, which has no metrizable support.*

This theorem disproves some already published claim. Our construction is based on two auxiliary results on the structure of the space $P(K)$, for K being Corson compact.