

### Introduction to Logic: Worksheet 3

1. Which derivation rules are used in the following derivations? Specify the appropriate justifications.

a)

1	A & B
2	B
3	A

b)

1	D
2	(D $\vee$ E) $\equiv$ F
3	D $\vee$ E
4	F

c)

1	P $\supset$ $\sim$ Q	
2	Q	
3	<table style="border-collapse: collapse;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">P</td> </tr> </table>	P
P		
4	<table style="border-collapse: collapse;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">Q</td> </tr> </table>	Q
Q		
5	<table style="border-collapse: collapse;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;"><math>\sim</math> Q</td> </tr> </table>	$\sim$ Q
$\sim$ Q		
6	$\sim$ P	

d)

1	S $\supset$ $\sim$ U		
2	R $\vee$ U		
3	R $\supset$ (S & T)		
4	<table style="border-collapse: collapse;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">S</td> </tr> </table>	S	
S			
5	<table style="border-collapse: collapse;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">U</td> </tr> </table> </td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">U</td> </tr> </table>	U
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9	<table style="border-collapse: collapse;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">R</td> </tr> </table>	R	
R			
10	<table style="border-collapse: collapse;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">R</td> </tr> </table>	R	
R			
11	<table style="border-collapse: collapse;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">R</td> </tr> </table>	R	
R			
12	R		
13	R		
14	S & T		
15	S		
16	S $\equiv$ R		

21 marks

2. Derive the following SL sentences from the specified assumptions. Specify the appropriate justifications.

- a) ' $\sim \sim$  A' from 'A'.
- b) ' $P \supset (Q \supset R)$ ' from ' $(P \ \& \ Q) \supset R$ '
- c) 'D' from ' $(B \vee C) \equiv (B \equiv D)$ ' and 'B'
- d) ' $A \supset B$ ' from ' $\sim A \vee B$ '
- e) ' $S \equiv T$ ' from ' $(S \vee U) \supset T$ ' and ' $T \supset S$ '

29 marks