Partial Differentiation: Extra Practice

In the lectures we went through Questions 1, 2 and 3. But I have plenty more questions to try!

Find $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ for the following functions:

1. $f(x, y) = (x^2 - 1)(y + 2)$
2. $f(x, y) = e^{x+y+1}$
3. $f(x, y) = e^{-x} \sin(x + y)$.

Solutions

1. First,

$$\begin{aligned}
\frac{\partial f}{\partial x} &= \frac{\partial}{\partial x}\left[(x^2-1)(y+2)\right] \\
&= (y+2)\frac{\partial}{\partial x}\left[(x^2-1)\right] \\
&= (y+2)(2x) \\
&= 2x(y+2).
\end{aligned}$$

Similarly,

$$\begin{aligned}
\frac{\partial f}{\partial y} &= \frac{\partial}{\partial y}\left[(x^2-1)(y+2)\right] \\
&= (x^2-1)\frac{\partial}{\partial x}\left[(y+2)\right] \\
&= (x^2-1)\cdot 1 \\
&= (x^2-1).
\end{aligned}$$

2. We can start by observing that

$$e^{x+y+1} = e^x e^y e.$$

So

$$\begin{aligned}
\frac{\partial f}{\partial x} &= \frac{\partial}{\partial x}\left(e\, e^x e^y\right) \\
&= e\, e^y \frac{\partial}{\partial x}\left(e^x\right) \\
&= e\, e^y (e^x) \\
&= e^{x+y+1}.
\end{aligned}$$

Similarly,

$$\begin{aligned}\frac{\partial f}{\partial y} &= \frac{\partial}{\partial y}\left(e\, e^{x} e^{y}\right)\\ &= e\, e^{x}\frac{\partial}{\partial y}\left(e^{y}\right)\\ &= e\, e^{x}(e^{y})\\ &= e^{x+y+1}.\end{aligned}$$

3. Using the Product Rule,

$$\begin{aligned}\frac{\partial f}{\partial x} &= \frac{\partial}{\partial x}\left(e^{-x}\right)\sin(x+y) + e^{-x}\frac{\partial}{\partial x}\left(\sin(x+y)\right)\\ &= -e^{-x}\sin(x+y) + e^{-x}(1\cdot\cos(x+y))\\ &= e^{-x}\left(\cos(x+y) - \sin(x+y)\right).\end{aligned}$$

Similarly,

$$\begin{aligned}\frac{\partial f}{\partial y} &= \frac{\partial}{\partial y}\left(e^{-x}\right)\sin(x+y) + e^{-x}\frac{\partial}{\partial y}\left(\sin(x+y)\right)\\ &= 0 + e^{-x}(1\cdot\cos(x+y))\\ &= e^{-x}\cos(x+y).\end{aligned}$$

Further practice questions

Here are some more practice questions you can try. These may be useful for exam revision.

Find $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ for the following functions:

1. $f(x, y) = (xy - 1)^2$
2. $f(x, y) = \dfrac{1}{x + y}$
3. $f(x, y) = \ln(x + y)$
4. $f(x, y) = \sin^2(x - 3y)$
5. $f(x, y) = x^y$.

Also, find $\dfrac{\partial f}{\partial x}$, $\dfrac{\partial f}{\partial y}$ and $\dfrac{\partial f}{\partial z}$ for the functions

1. $f(x, y, z) = 1 + xy^2 - 2z^2$
2. $f(x, y, z) = x - \sqrt{y^2 + z}$
3. $f(x, y, z) = e^{-xyz}$.

Solutions will be typed up soon!