

Magnetic properties of materials

John JL Morton

Part 1. Introduction to magnetism

1.1 Origins of magnetism

The phenomenon of magnetism was most likely known by many ancient civilisations, however the first recorded description is from the Greek Thales of Miletus (ca. 585 B.C.) who writes on the attraction of loadstone to iron. By the 12th century, magnetism is being harnessed for navigation in both Europe and China, and experimental treatises are written on the effect in the 13th century. Nevertheless, it is not until much later that adequate explanations for this phenomenon were put forward: in the 18th century, Hans Christian Ørsted made the key discovery that a compass was perturbed by a nearby electrical current. Only a week after hearing about Ørsted's experiments, André-Marie Ampère, presented an in-depth description of the phenomenon, including a demonstration that two parallel wires carrying current attract or repel each other depending on the direction of current flow. The effect is now used to define the unit of current, the *amp* or *ampere*, which in turn defines the unit of electric charge, the *coulomb*.

1.1.1 Ampère's Law

Magnetism arises from *charge in motion*, whether at the microscopic level through the motion of electrons in atomic orbitals, or at macroscopic level by passing current through a wire. From the latter case, Ampère's observation was that the magnetising field H around any conceptual loop in space was equal to the current enclosed by the loop:

$$I = \oint H dl \quad (1.1)$$

By symmetry, the magnetising field must be constant if we take concentric circles around a current-carrying wire. Integrating around such a circular loop of radius r :

$$(1.2)$$

1.2 Magnetic fields and moments

In magnetism, the analogous field to the electric field E , is the *magnetising* field H , which has units A/m. In practice, we often use the field B , which we call the magnetic field and has the units Wb/m² or Tesla¹. In a vacuum, H is related to B by the permeability of free space, μ_0 which has the defined value $4\pi \cdot 10^{-7}$.

(1.3)

In the same way that electric dipoles minimise their potential energy by aligning with an electric field, magnetic dipoles, or *magnetic moments*, have some potential energy in a magnetic field which is minimised when they align with the field. In a magnetic field B , the potential energy of a magnetic moment m is:

(1.4)

A circular loop of current has an effective magnetic moment given by the product of the loop area A and the current I :

(1.5)

1.3 Magnetic moments of electrons

We know that a current loop produces a magnetic moment — so what is the magnetic moment of an orbiting electron? An electron travelling at velocity v around an orbit of radius r corresponds to a current of $ev/2\pi r$ around a loop of area πr^2), which has a magnetic moment:

(1.6)

Noticing that this is proportional to vr , we can express this in terms of the electron's angular momentum $\Pi_e = m_e vr$ were m_e is the mass of the electron²:

(1.7)

Despite this simple argument, it is very close to the precise quantum mechanical description, which only differs by an additional constant factor, which is called the *g-factor*, and has the approximate value of $g_e \approx 2$:

$$m = g_e \frac{e\Pi_e}{2m_e} \quad (1.8)$$

¹There is a fair bit of inconsistency in the names given to these fields: H is sometimes called the magnetic field, B is sometimes called the *flux density*. The key is always to look at the units to see what is being quoted.

²Not to be confused with the magnetic moment m

So, depending on the electron angular momentum, it will have some magnetic moment.

Regardless of whether it is orbiting an atom, electrons possess an *intrinsic* angular momentum which we call its *spin*. This is a quantum mechanical property, labeled with the quantum number S , which can take one of two values: $S = 1/2$ or $-1/2$. We often call the $S = 1/2$ state, *spin-up*, and the $S = -1/2$, *spin-down*. The resulting angular momentum along a particular direction is $\hbar S$. We use Eq. 1.8 to calculate the magnetic moment from this angular momentum:

$$m = g_e \frac{e\hbar}{2m_e} S \quad (1.9)$$

Thus, the magnetic moment of an electron depends on the g-factor g_e , the spin state S , and a bunch of fundamental constants which are usually lumped together and named μ_B , the *Bohr magneton*:

$$(1.10)$$

The Bohr magneton is thus defined as

$$\mu_B = \frac{e\hbar}{2m_e} = 9.27 \cdot 10^{-24} \text{ Am}^2. \quad (1.11)$$

Key result

All electrons possess a magnetic moment, whose value is (approximately) $\pm\mu_B$.

The total angular momentum J of an electron in a general atomic orbital has two parts: the intrinsic spin S , and the angular momentum of the particular orbital it is in L . In practice, the orbital angular momentum L is locked into the crystal structure and does not respond to an applied magnetic field (this is known as *quenching* of the orbital moment). Such environmental effects can be bundled into the electron g -factor, so this takes values that will differ slightly from 2, depending on the precise environment of the electron.

1.4 Magnetic moments of atoms

Having established that individual electrons have magnetic moments, we now look at how these may lead to magnetic moments in atoms. Electrons occupy well-defined orbital states in atoms (e.g. 1s, 2s, 2p etc.). Although, electrons are fermions and therefore cannot occupy exactly the same state, each of these orbital states can accommodate *two* electrons because the electrons can be *spin-up* or *spin-down*. If one electron occupies an atomic orbital, it

1. Introduction to magnetism

Ion	Mn ²⁺	Fe ²⁺	Co ²⁺	Ni ²⁺	Cu ²⁺	Zn ²⁺
Z	25	26	27	28	29	30
m	$5\mu_B$	$4\mu_B$	$3\mu_B$	$2\mu_B$	μ_B	0
S	$5/2$	2	$3/2$	1	$1/2$	0

Table 1.1: Magnetic moment of important divalent transition metals

is free to take either spin state, however if two electrons occupy the orbital, they must take opposing spin states and thus their magnetic moments cancel out. The total spin state of the atom can then be determined by considering the net spin of the constituent electrons. For example, hydrogen has one electron in the $1s$ state, so it is net $S = 1/2$, or in other words magnetic moment $\sim \mu_B$. On the other hand, helium has two electrons in the $1s$ state ($1s^2$) which must anti-align, giving net $S = 0$, or zero net moment.

For larger atoms we use one of Hund's rules to work out how electrons occupy atomic orbitals: "in each electron shell the spins align to produce the largest magnetic moment". For example, there are five d -shell orbitals. These are filled sequentially with one electron in each orbital state, such that an atom/ion with five d -shell electrons (e.g. Mn²⁺) is $S = 5/2$, or a magnetic moment $5\mu_B$. Subsequent electrons added to the d -shell must then pair up with the others, such that the filled 10-electron d -shell has zero net moment.

Fe has electron configuration $1s^2 2s^2 2p^6 3s^2 3p^6 3d^6 4s^2$. The Fe²⁺ ion loses the two $4s$ electrons. All filled orbitals can be ignored as they must have no net moment. This leaves $3d^6$, which fills as ($\uparrow\downarrow, \downarrow, \downarrow, \downarrow, \downarrow$) and has net moment $4\mu_B$ ($S = 2$). The Fe³⁺ ion loses an additional d electron and has net moment $5\mu_B$ ($S = 5/2$).

1.5 Magnetic susceptibility

In the last course on the Electric and Optical Properties of Materials we saw how a material could be made up of many permanent electric dipoles p . The potential energy of such dipoles in an electric field E was: $U = -p \cdot E$, and so the application of an electric field can produce some alignment of the dipoles to give an overall electric polarisation P in the material. Thus, the electric displacement field D within a material arises from a combination of an applied electric field E , and a contribution from the electric polarisation of the material P .

$$D = \epsilon_0 E + P \tag{1.12}$$

Given the polarisation was itself proportional to E :

$$D = \epsilon_0 E + \chi_e \epsilon_0 E = \epsilon_0(1 + \chi_e)E = \epsilon_0 \epsilon_r E \quad (1.13)$$

The situation for magnetism is entirely analogous. Materials may also possess *magnetic moments*, m , which sum up to give an overall magnetisation M in the material, per unit volume:

$$(1.14)$$

Thus, the magnetic field in a material is due to contributions from both H , and this magnetisation, M :

$$(1.15)$$

In many cases³, this magnetisation is proportional to the applied magnetising field H , and so we define the magnetic susceptibility χ , as the magnetisation achieved given a certain magnetising field:

$$(1.16)$$

It follows that:

$$(1.17)$$

thus defining the relative permeability μ_r . These analogous quantities in electricity and magnetism are summarised in Table 1.2.

³Note that there are important cases where the magnetisation M is not proportional to the applied field H , for example a permanent magnet can have magnetisation even in the absence of any applied field.

1. Introduction to magnetism

E Electric field (V/m)	H Magnetising field (A/m)
D Displacement field (C/m ²) $D = \epsilon E$	B Magnetic field (Wb/m ²) $B = \mu H$
P Polarisation (C/m ²) $P = \chi_e \epsilon_0 E$	M Magnetisation (A/m) $M = \chi H$
p Electric dipole (Cm) $U_p = -p \cdot E$	m Magnetic moment (Am ²) $U_m = -m \cdot B$
ϵ_0 vacuum permittivity (F/m)	μ_0 vacuum permeability (H/m)
ϵ_r relative permittivity $\epsilon_r = 1 + \chi_e$	μ_r relative permeability $\mu_r = 1 + \chi$
V Voltage (V)	I Current (A)
q Charge (Coulomb, C)	Φ Magnetic Flux (Weber, Wb)
C Capacitance (Farad, F) $C = q/V$	L Inductance (Henry, H) $L = \Phi/I$

Table 1.2: Analagous quantities in electricity and magnetism