Magnetic properties of materials
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Part 2. Types of magnetism (or $-1 \leq \chi < \inf$)

Different materials respond to applied magnetic fields in different ways. An instructive way to see this dependence is to look at the magnetic susceptibility, χ. In this chapter we will examine the different types of magnetism, the origin of this different behaviour, and the implications on the susceptibility. We will begin by looking at *diamagnetism* in materials with no permanent magnetic moments, then *paramagnetism* in materials with magnetic moments that do not interact with each other. Finally, we consider materials where there is strong interaction between the magnetic moments, leading to *ferrormagnetic*, *anti-ferromagnetic*, or *ferrimagnetic* behaviour.

2.1   Diamagnetism

Diamagnetism describes the case where the magnetisation in a material *opposes* the applied field H, reducing the magnetic field in the material from what would be there if the material was absent, or in other word, expelling some of the applied field. An extreme case is that of superconductors which we saw in the previous course are perfect diamagnets, expelling entirely the field H, thus achieving a magnetisation of $-H$, and having a magnetic susceptibility $\chi = -1$.

In fact, all materials must have some element of diamagnetism due to Lenz's Law, which states that "An induced current is always in such a direction as to oppose the motion or change causing it". Thus, under an applied magnetic field, electrons in atomic orbitals will slightly adjust their orbit in such a way to create current loops that oppose the applied field. The magnetic moments of each of these current loops add up to give an overall magnetisation in the material under a magnetising field, H.

Consider a single electron in orbit of radius r around an atom. Under an applied magnetic field B, it will orbit at an angular frequency $\omega = eB/2m_e$, counteracting the applied field, where m_e is the electron mass. This appears as a current:

$$(2.1)$$

around a loop of area πr^2, leading to a magnetic moment:

$$(2.2)$$

An atom with atomic number Z will possess Z such electrons, with mean some orbital radius $\langle r^2 \rangle$. The total magnetisation will therefore be:

$$ \text{(2.3)} $$

where N is the concentration of atoms (m^{-3}). Therefore, the diamagnetic susceptibility is:

$$ \text{(2.4)} $$

As an example we can estimate the diamagnetic susceptibility of water, adapting the formula above for use with a molecule. The density of water is 1000 kgm^{-3}, so there are about $3 \cdot 10^{28}$ molecules/m^{-3}, with a total of 10 electrons per molecule. We can estimate the mean atomic orbital to be the Bohr radius (0.5 Å $= 5 \cdot 10^{-11}$ m). These numbers give an estimated susceptibility of $-7 \cdot 10^{-6}$, which is very close to the measured value ($-9 \cdot 10^{-6}$). Typical magnetic susceptibilities for diamagnetic materials are in the region of $\chi \sim -10^{-5}$.

2.2 Paramagnetism

Although we stated above that all materials exhibit some diamagnetism, this may be negligible compared to a positive magnetic susceptibility arising from the magnetic moments of unpaired electrons aligning themselves with the applied field. This is known as *paramagnetism*.

Consider the behaviour of magnetic moments, possessed by certain atoms and ions, in an applied magnetic field. The energy will be minimised if one of the states (say the *spin-down* state) aligns with the magnetic field, leaving the other state (the *spin-up*) aligned against it. Thus, for each atomic orbital, the energies of the *spin-down* or *spin-up* state shift by $-\mu_B B$ and $+\mu_B B$ respectively, as shown in Figure 2.1.

At some finite temperature the electron will be found with greatest probability in the lower energy state, as given by Boltzmann statistics, where the probability of being in a state is proportional to $\exp(-U/k_B T)$ if U is the energy of the state. As the magnetic field is increased, the probability of electrons being in the spin-down state increases, therefore more moments are aligned with the field than against it, and the materials acquires more magnetisation. At zero applied field (or very high temperature), the probability of being in the spin-up or spin-down state is equal, therefore the moments are equally oriented with and against the field, and the material has no magnetisation.
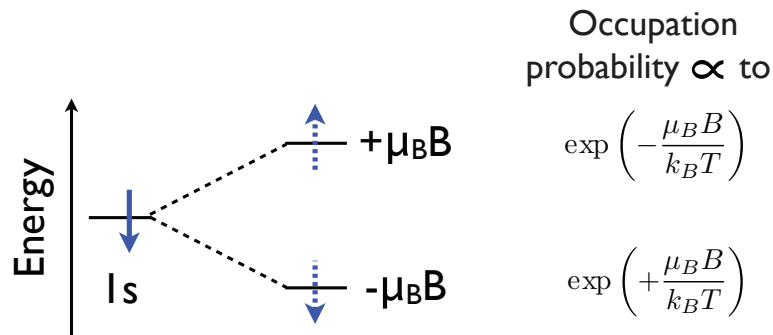
Figure 2.1: Energies of atomic orbitals split in an applied magnetic field, such that the spin-up and spin-down state no longer have the same energy.

We can thus derive an expression for the magnetisation (and suscepti-bility) by looking at the number of spin-up vs. spin down electrons. The number of spin-down electrons per unit volume will be:

$$(2.5)$$

where N is the total number of atoms (which possess a magnetic moment) per unit volume in the material. Similarly:

$$(2.6)$$

The total magnetisation of the material will be given by the excess of spin-down moments

$$(2.7)$$

$$(2.8)$$

In the limit of high temperature / small magnetic field, we can approximate[1] $\tanh(x)$ to x, and so

$$(2.9)$$

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[1]As a good rule of thumb, if the temperature in Kelvin is greater than the magnetic field in Tesla, this approximation is valid.

Thus, the susceptibility is:

$$(2.10)$$

This is the *Curie Law* for paramagnetic susceptibility. The derivation above assumes each atom contains one unpaired electron, though we can easily generalise it by substituting a general magnetic moment m for the Bohr magneton μ_B:

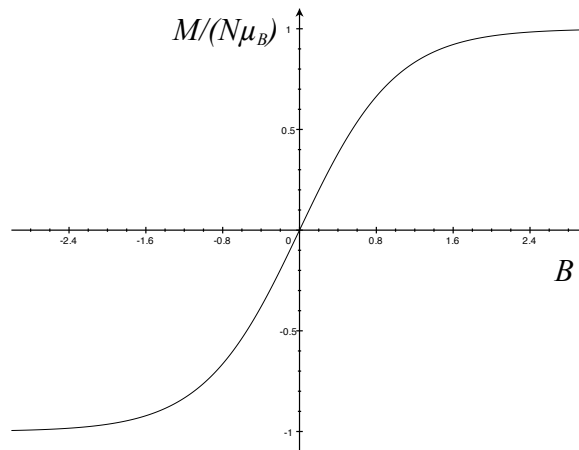$$\chi = \frac{Nm^2\mu_0}{k_BT} \qquad (2.11)$$

Figure 2.2: Magnetisation M of a paramagnetic material as a function of the applied magnetic field B, at a temperature of about 0.7 K.

2.3   Pauli paramagnetism

In addition to unpaired electrons in atoms, an important contribution to paramagnetism can be found from the conduction electrons in metals. This is known as *Pauli paramagnetism*, in contrast to the *Curie paramagnetism* described above. This effect is illustrated in Figure 2.3. Consider the density of states for electrons in a solid, and how electrons occupy these states up to Fermi energy, E_F. We can alternatively represent the spin-up and spin-down electrons separately, though in zero applied field the density of states would be the same for both.

When a magnetic field B is applied, these densities of states shift up and down by the energy of the spin interacting with the field ($\pm\mu_B B$), with the result that a number of the spin-up electrons must flip to minimise the total
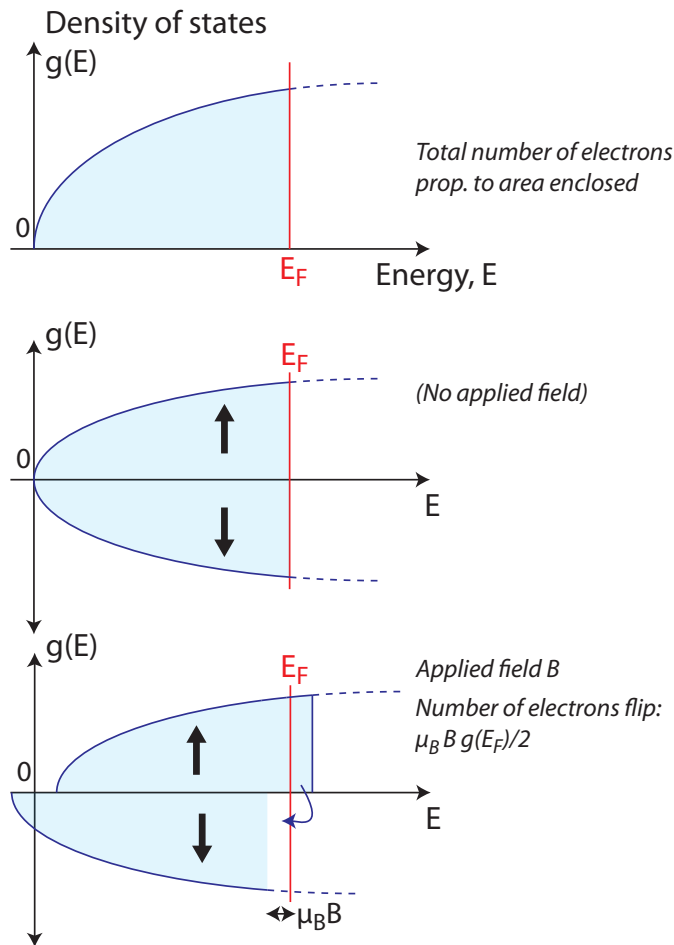
Figure 2.3: Pauli paramagnetism: magnetisation arising from conduction electrons flipping to align with an applied B field.

energy. This number is given by the area enclosed in the plot of density of states vs. energy, and is:

$$\tag{2.12}$$

where $g(E_F)$ is the density of states at the Fermi energy $= 3\mathcal{N}/2E_F$ and $\mathcal{N}$ is the total number of conduction electrons. The net change in moment per spin that flips is $2\mu_B$, so the resulting magnetisation is:

$$\tag{2.13}$$

where N is the concentration of conduction electrons. The resulting param-

agnetic susceptibility is:

$$\tag{2.14}$$

If we rewrite the Fermi energy in terms of the Fermi temperature ($E_F = k_B T_F$), we see that this has essentially the same form as the Curie paramagnetic susceptibility (Equation 2.10), but with the actual temperature T replaced with the Fermi temperature:

$$\tag{2.15}$$

Let's take Magnesium as an example: the Fermi energy is 7 eV and the density is 1700 kg/m^3, corresponding to $4.2 \cdot 10^{28}$ atoms/m^{-3}, each donating 2 'free' electrons. Using these numbers we get a predicted susceptibility of $1.2 \cdot 10^{-5}$, which is also the measured value.

2.4 The exchange interaction

So far we have assumed no interaction between the magnetic moments in the material, with the result that when an external magnetic field is removed, there is nothing to keep the moments aligned and there is no residual magnetisation. However, we know that some materials exhibit a permanent magnetisation after the field is removed. This could be explained by an interaction between moments which favours their alignment — but what is the origin of such an interaction?

The magnetic dipolar interaction energy of two moments m separated by a distance r is approximately $\mu_0 m^2 / 4\pi r^3$. If we evaluate this using μ_B for the moment and 3 Å as the separation, the interaction strength is 3×10^{-25} J, or in other words equivalent to 0.02 Kelvin. This interaction is evidently far too weak to account for the fact that permanent magnets exist at (e.g.) room temperature.

Instead, we have to turn back to quantum mechanics and the Pauli exclusion principle which forbids two electrons from occupying exactly the same state. In a solid, the atomic orbitals overlap and so if two electrons on neighbouring atoms are to occupy the same state in in space, they must take opposite spin. This is known as the *exchange interaction*, and although it affects how magnetic moments align, it is fundamentally an *electrostatic* interaction because it arises from the electrostatic forces that cause electron wavefunctions to overlap. Unlike the dipolar interaction, the energy of the exchange interaction can be large, many times the thermal energy at room temperature.

We've discussed the *negative* exchange interaction which favours the *anti*-alignment of neighbouring moments. In general, if we also consider the conduction electrons present in the solid, the sum of the different exchange interactions between different moments can be complicated, and result even in a *positive* exchange interaction, favouring the *alignment* of neighbouring magnetic moments.

The exchange energy between two spins i and j is:

$$U_{ex}(ij) = -2JS_i \cdot S_j \tag{2.16}$$

for an exchange interaction J. Using Equation 1.10, we can also write this in terms of moments:

$$U_{ex}(ij) = -m_i \cdot m_j \frac{2J}{g^2\mu_B^2} \tag{2.17}$$

We can sum up the exchange interaction between spin i and all of its neighbours, taking an average moment of its neighbours $\bar{m}$ (which is the total magnetisation of the material divided by the concentration of atoms $\bar{m} = M/N$), and an average exchange coupling $\bar{J}$:

$$U_{ex}(ij) = -m_i \cdot \frac{2\bar{J}}{g^2\mu_B^2 N} M \tag{2.18}$$

This bears a strong resemblance to the energy of a moment m_i in a magnetic field ($U = -m_i \cdot B$). In other words, magnetic moments in a solid can also feel a very strong interaction from its neighbours, which appears like an effective magnetic field, called the *internal* or *molecular* magnetic field. It is important to stress this is not a real magnetic field in any sense, rather it is simply a tool for treating the overall effective exchange interactions from all of the neighbouring moments. The more aligned the neighbours are, the stronger this internal magnetic field, hence we can give it the value λM, where M is the magnetisation of the material, and λ is a constant, proportional to the strength (and sign!) of the exchange interaction J.

Key result
In certain materials, a strong quantum mechanical interaction exists between spins which can be treated as a large *internal* magnetic field proportional to the magnetisation of the material: $B_{\text{int}} = \lambda M$.

2.5 Ferromagnetism

Above, we derived an approximate expression for the paramagnetic susceptibility $\chi = C/T$ (where C is the Curie constant $N\mu_B^2\mu_0/k_B$). Hence, given

a magnetic field B, the magnetisation of the material is:

$$(2.19)$$

If this magnetic field is a combination of an applied magnetic field B_{app} and the effective internal magnetic field B_{int} arising from the exchange interaction, we can write this as:

$$(2.20)$$

Rearranging for M:

$$(2.21)$$

Allowing us to extract the new susceptibility for this material, given the presence of a positive exchange interaction between the moments causing them to align:

$$\chi = \frac{C}{T - C\lambda/\mu_0} \qquad (2.22)$$

Often this is written in terms of a critical temperature, θ_W known as the *Curie-Weiss temperature* when this susceptibility diverges:

$$(2.23)$$

This is the *Curie-Weiss* law. The Curie-Weiss temperature for a material can be estimated, given a knowledge of the internal magnetic field constant λ:

$$\theta_W = \frac{N\mu_B^2\lambda}{k_B} \approx \frac{\bar{J}}{2k_B} \qquad (2.24)$$

In other words, the Curie-Weiss temperature corresponds to the point at which thermal energy $k_B T$ becomes comparable with the mean exchange coupling energy $\bar{J}$. When the temperature is at or below θ_W, the susceptibility can be infinite, resulting in a magnetisation even in the absence of a magnetising field. The permanent magnets that we generally use therefore have a Curie-Weiss temperature greater than 300 K. Below θ_W, the magnetisation can depend on a number of factors, including the *domain structure* of the material, which we will discuss later in the course.

2.6   Anti-ferromagnetism

Exchange coupling can also be negative (very common, especially for non-metals), leading to anti-alignment between neighbouring spins and a negative effective magnetic field. Just as for ferromagnets, there is a critical ordering temperature, which is called the *Néel temperature* for anti-ferromagnets, which again corresponds to the point at which the magnitude of the (negative) exchange energy becomes greater than thermal energy, and spins anti-align. Above this temperature, we can derive a susceptibility, following a method very similar to the one above, except we must consider two separate and intertwined sub-lattices. We shall just quote the result, which is analogous to that for ferromagnetism:

$$\chi = \frac{C}{T + \theta_N} \qquad (2.25)$$

Below the Néel temperature, the susceptibility will depend on the orientation of the field to the crystal, but in general will fall back towards zero as the temperature is decreased and the material develops a completely ordered (anti-ferro)magnetic state with zero net magnetisation.

The temperature dependence of this magnetic susceptibility is shown in Figure 2.4, along with that for ferromagnets and paramagnets.

2.6.1   Anti-ferromagnetism and superexchange

Within many compound materials, metal ions are separated by nonmetals, such as oxygen. An example is nickel oxide: $Ni^{2+}O^{2-}$. Rather than leading to isolated magnetic moments and thus to paramagnetic behaviour, the propeller-shaped $2p$ orbital oxygen can *mediate* an exchange interaction between nickel ions, in what is called a *superexchange interaction* (because it happens via another ion). As illustrated in Figure 2.5, this is anti-ferromagnetic, and is strongest when the M-O-M angle is close to 180°.

The Néel temperature for various oxides are shown in Table 2.1 below. The stronger the degree of covalency, the greater the overlap between the O and metal orbitals, the stronger the exchange interaction, and the higher the magnetic ordering temperature.
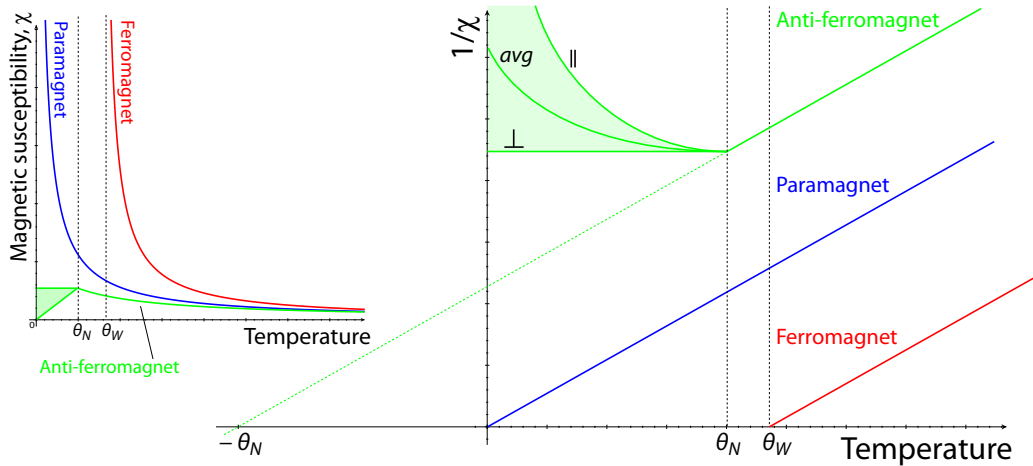
Figure 2.4: Magnetic susceptibility as a function of temperature for different types of magnetic material (not shown is diamagnetic susceptibility which is negative and temperature independent). It is convenient to plot the reciprocal of the susceptibility against temperature: each type of material should show a linear dependence, but the intercept for $1/\chi = 0$ should be $T = 0$ for a paramagnet, $T > 0$ for a ferromagnet (the Curie Weiss temperature θ_w) and $T < 0$ for an anti-ferromagnet (the Néel temperature θ_N).
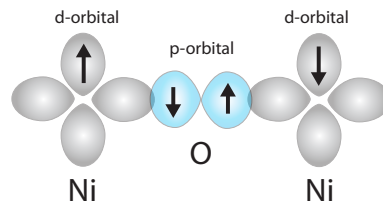
Figure 2.5: The anti-ferromagnetic superexchange interaction between two Ni ions, mediated by the O p-orbital.

Compound	Θ_N
MnO	116 K
FeO	198 K
CoO	293 K
NiO	523 K

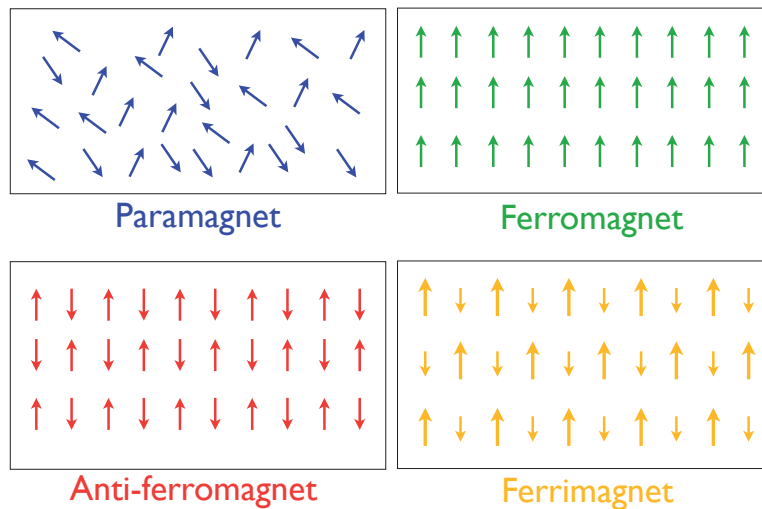Table 2.1: Néel temperature for various metal oxides

Figure 2.6: A graphical summary of the different types of magnetic phenomena.

2.7 Ferrimagnetism

Some materials exhibit anti-ferromagnetic coupling between atoms of *unequal* moments, so that even in the maximally ordered magnetic state, there is an overall magnetisation — this is known as *ferrimagnetism*. This, and other types of magnetic phenomena are summarised in Figure 2.6.

2.7.1 Types of ferrimagnetic materials

A classic example of a ferrimagnetic material is 'loadstone' or Fe_3O_4, which is composed of $(Fe^{2+}O).(Fe_2^{3+}O_3)$. The Fe^{2+} ion $(3d^6)$ has $m = 4\mu_B$ while the Fe^{3+} ion $(3d^5)$ has $m = 5\mu_B$. The crystal structure is similar to spinel $(MgO.Al_2O_3)$, as shown in Figure 2.7, and it belongs to a larger class of *ferrite* materials which consist of replacing the Fe^{2+} with some other divalent ion such as Mn, Co, Ni, Cu or Zn.

Within this structure, anti-ferromagnetic interactions are via superexchange, described above. The B-O-B bond angle is 90°, the A-O-A angle is $\approx 80°$ and the A-O-B angle is $\approx 125°$. The superexchange interaction is therefore strongest across the A-O-B bond, so that magnetic moments on A sites anti-align with those on the B sites.

There are two principal arrangements of the 8 divalent and 16 trivalent metal ions onto the 8 tetragonal A sites and 16 octahedral B sites of the spinel structure. The first, called *normal spinel* has divalent metal ions on A

Layers at heights 0, a/4, a/2, 3a/4   ● A   *At height a/8 above layer*
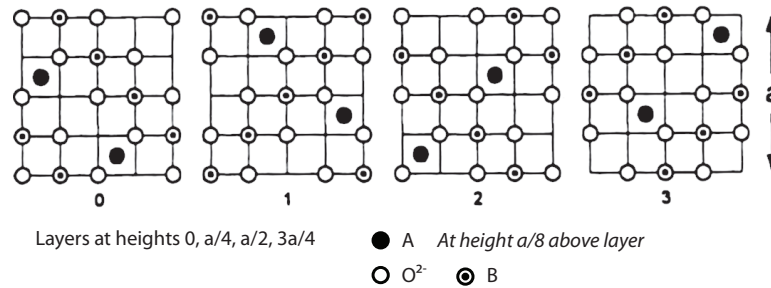
○ O^{2-} ◉ B

Figure 2.7: The spinel structure (layer by layer)

sites and trivalent ions on B sites. This is sometimes written $(M^{2+})[M^{3+}]O_4$, where the brackets indicate the $A()$ and $B[]$ sites.

Inverse spinel has half the trivalent ions on A sites, the other half on B sites, and all the divalent ions also on B sites. This can be written as $(M^{3+})[M^{2+}M^{3+}]O_4$.

Most simple ferrites (such as Fe_3O_4) have this inverse spinel structure. In this case, the anti-ferromagnetic superexchange interaction between ions on sites A and B is such that the magnetic moments of the trivalent ions (e.g. Fe_3) anti-align.

Another important interaction in inverse spinel ferrites is *double exchange*, where two cations of different charge occupy the same type of lattice site and are thus capable of exchanging an electron (see Figure 2.8). The electron that hops from the Fe^{2+} ion must be the one that is anti-aligned with the other five electrons on the ion (to preserve Hund's rules) — say it is spin-up. It must displace a spin-up electron on the oxygen, which then hops onto the Fe^{3+} ion, which it can only do if those electrons are all spin-down, or in other words the same as the electrons on the Fe^{2+} ion. This leads to a ferromagnetic interaction. A similar interaction is possible with other ions, e.g. Mn^{2+} and Mn^{3+}.
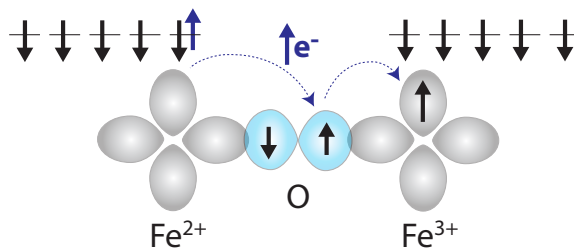

Figure 2.8: The ferromagnetic double exchange interaction between two cations of the same metal species with differing charge.

Therefore, though the trivalent ions are anti-ferromagnetically coupled to each other, half of them are ferromagnetically coupled to the divalent ions (see Figure 2.9.) The net moment of the ferrite is then due to the divalent ion only, which in the case of Fe^{2+} is $m = 4\mu_B$. This is consistent with the experimentally measured value of $4.08\mu_B$.
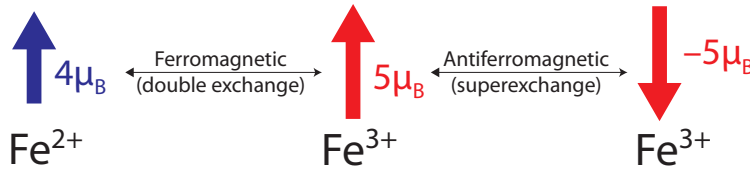

Figure 2.9: A summary of the interactions in Fe_3O_4, leading to ferrimagnetic behaviour.

2.8   Magnetic domains

Given the arguments above, we would expect a ferromagnetic material to be permanently magnetised below the ordering temperature θ_W, due to the ferromagnetic exchange interaction. However, we know that it is possible to de-magnetise ferromagnetic material so there is no overall magnetisation. The magnetisation in ferromagnetic materials is critically dependent on the formation and behaviour of *domains* of magnetic ordering.

This concept of domains was introduced by Pierre-Ernest Weiss in 1907 and can be understood from arguments based on minimising the sum of the various energies present, in addition to the exchange energy.

2.8.1   Magnetostatic energy

The key energy that dictates why magnetic domains must form is the magnetostatic energy arising from the stray fields both inside and outside the material. As a reminder, the energy stored in the magnetic field per unit volume is:

$$U = \frac{1}{2}\mu_0 H^2 \tag{2.26}$$

As illustrated in Figure 2.10, this magnetostatic energy can be minimised through the formation of magnetic domains within the material which align against each other to minimise the external magnetic field.
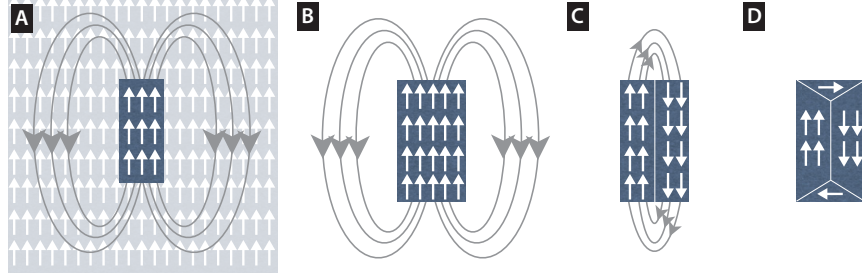
Figure 2.10: There is considerable magnetostatic energy (A) within magnetic material and (B) outside the material if it is completely magnetised with all moments aligned. (C,D) This energy is reduced by the formation of magnetic domains which align against each other.

2.8.2 Anisotropy energy

There are other energy considerations that will dictate the size and preferred orientation of domains. The first of these is the *anisotropy energy* which arises because in crystalline lattices, the energy is lower for magnetisation parallel to certain crystallographic directions: these are called *easy axes* of magnetisation, versus *hard axes* of magnetisation. There is an energy cost for a magnetisation aligned away from an easy axes, towards a hard axis.

An example is the magnetisation of an Fe single crystal shown in Figure 2.11, where $\langle 100 \rangle$ is the easy axis). Co has an easy axis of $\langle 0001 \rangle$ at room temperature, while Ni has easy axes along $\langle 111 \rangle$.

The anisotropy energy is expressed as a power series expansion with form depending on crystal structure. For cubic crystals, the anisotropy energy per unit volume is:

$$E_K = K_1(\alpha_1^2 \alpha_2^2 + \alpha_2^2 \alpha_3^2 + \alpha_1^2 \alpha_3^2) + K_2(\alpha_1^2 \alpha_2^2 \alpha_3^2) + ... \tag{2.27}$$

Where $\alpha_i = \cos\theta_i$, and θ_i is the angle between the magnetisation and each of the $\langle 100 \rangle$ directions. The anisotropy energy is then characterised by the first and second order anisotropy constants: K_1 and K_2. The larger K_1, the larger the anisotropy energy, i.e. the difference in energy between magnetisation along the easy vs. hard axes. For $K_1 > 0$ magnetisation along $\langle 100 \rangle$ has lowest energy, while for $K_1 < 0$ magnetisation along $\langle 111 \rangle$ has lowest energy.

For crystal structures of lower symmetry where there is only one easy axis, the anisotropy energy can be written as $E_K = K_1 \sin^2\theta$, where θ is the angle of the magnetisation from the easy axis.
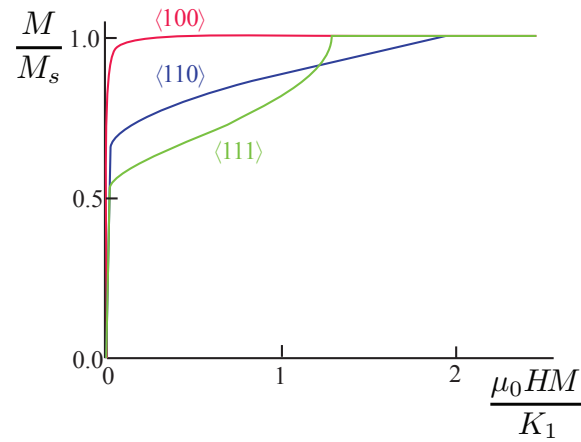
Figure 2.11: Magnetisation of an iron single crystal along different crystallographic directions. For a sufficiently large magnetising field, the saturation magnetisation M_s is always reached, however this occurs for a much smaller field when oriented along the $\langle 100 \rangle$ direction (easy axis).
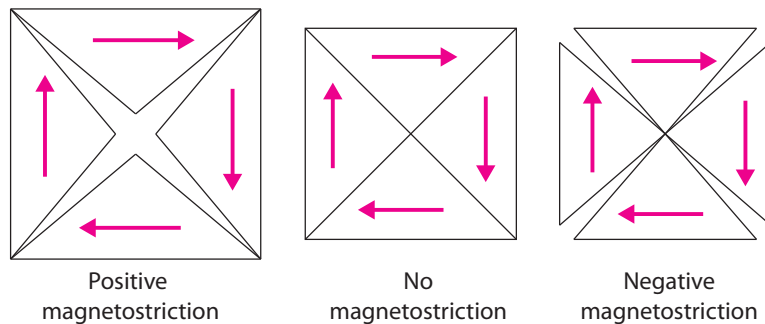

Figure 2.12: The effect of magnetostriction

2.8.3   Magnetoelastic energy

Finally, the dimensions of the magnetic material can change in an applied field, in an effect known as *magnetostriction*. Positive magnetostriction is defined as the case where domains expand in the applied field direction, while for negative magnetostriction they contract in the applied field direction. The inverse of this effect also occurs: an applied stress can alter the magnetic anisotropy of the material.

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2.8.4 Energy summary

In summary, we have i) the exchange energy which favours regions where spins are aligned with their neighbours, ii) the magnetostatic energy which favours breaking down these regions into small magnetic domains and iii) the anisotropy energy which favours the alignment of these domains along certain easy axes. Additional effects such as magnetostriction further add to the complexity of the problem, and all these contributions balance to form the eventual domain structure of a material.

2.9 Domain walls

Domains of magnetic alignment must be separated by walls where the moments gradually rotate from the orientation of one domain to the other, as shown in Figure 2.13. A positive exchange energy favours minimising the angle between the moments, and thus very wide wall, however, the anisotropy energy favours a minimum domain wall thickness so that more moments are aligned along easy axes. By considering both of these energies we will calculate an expression for domain wall thickness.
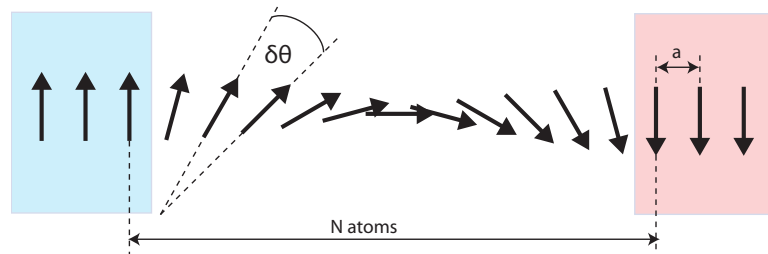

Figure 2.13: A domain wall of N spins separated by a which gradually rotate in magnetisation. Two adjacent spins differ in orientation by angle $\delta\theta$.

Consider a wall consisting of N spins separated by the interatomic distance a, as shown in the figure. Remembering that the exchange interaction between two neighbouring spins i and j is $U_{ex}(ij) = -2JS_i \cdot S_j$, we can write down the exchange energy between two spins, differing in orientation by $\delta\theta$:

$$(2.28)$$

Which, for small $\delta\theta$ approximates to:

$$(2.29)$$

If the two domains are anti-aligned, $\delta\theta = \pi/N$, and there are N such interactions along a chain, which has unit area a^2 (assuming a primitive cubic lattice). So, the exchange energy per unit wall area is:

$$ \tag{2.30} $$

which is minimised for the largest N, or widest domain wall. On the other hand, the anisotropy energy K per unit volume, leads to an energy of KNa per unit wall area, which is minimised by a narrow domain wall. Summing the two together:

$$ \tag{2.31} $$

And finding the minimum:

$$ \tag{2.32} $$

$$ \tag{2.33} $$

The total domain wall thickness $w = Na$:

$$ \tag{2.34} $$

Typical values are $a = 3$ Å, $K = 10^3$ to 10^5 J/m^3. Typical exchange energies can be derived from typical Curie temperatures (100 to 1000 K), or $J = 10^{-21}$ to 10^{-20} J, which give $w = 10$ to 300 nm. In other words, the domain walls are much smaller than the typical domain size of many tens to hundreds of microns.

The total energy of the domain wall limits the subdivision of domains into smaller and smaller domains:

$$ \tag{2.35} $$

2.9.1   Domain wall motion

When a magnetic field is applied, the sample acquires a magnetisation which is achieved by changing the relative size of the different domains. In other words, the domain wall moves, growing the size of the magnetically-favourable domain (see Figure 2.14). Eventually, for larger applied fields, the magnetic orientations of the domains will rotate (away from the easy axis) towards that dictated by the field.
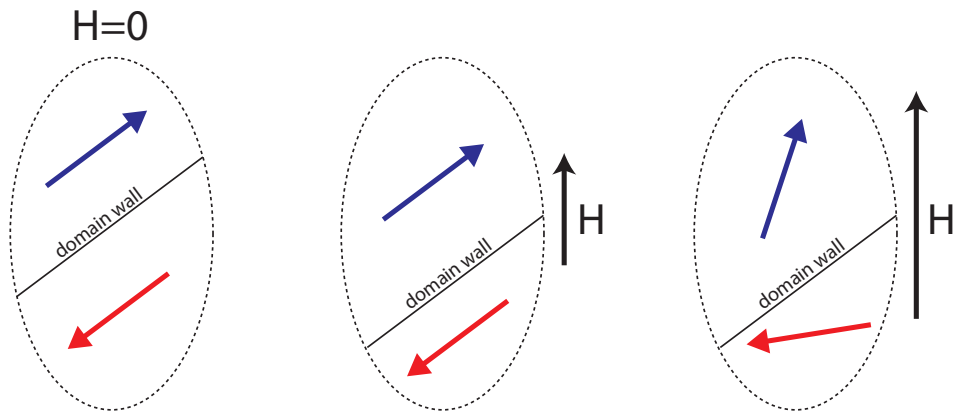
Figure 2.14: Possible effects of an applied magnetic field on domains within a region inside a magnetic material.

2.9.2 Domain wall pinning

The existence of permanent magnets tells us that domain wall motion is not simply reversible, and the original domain structure will not be recovered upon the removal of the magnetic field. An important factor in this is the pinning of domain walls at impurities and crystal defects, which restricts domain wall motion. As a simple example, consider a non-magnetic impurity atom in a chain of ferro-magnetically coupled atoms. The exchange energy falls off very quickly with distance, as it depends on the overlap of electron wavefunctions. The separation of two ferromagnetic atoms by even one impurity can be enough to essentially turn off the coupling between them, making this point ideal for a domain wall boundary (see Figure 2.15), and therefore a local potential energy well for a domain wall moving through the material.
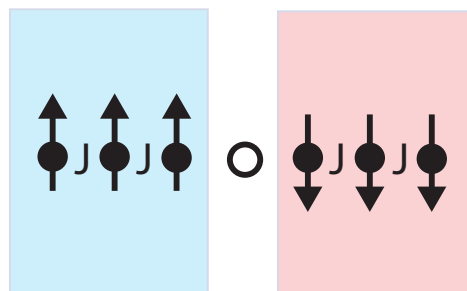

Figure 2.15: Non-magnetic impurities offer low-energy sites for domain walls by reducing the exchange energy between spins on either side of the domain wall. This helps restrict domain wall motion through a material.
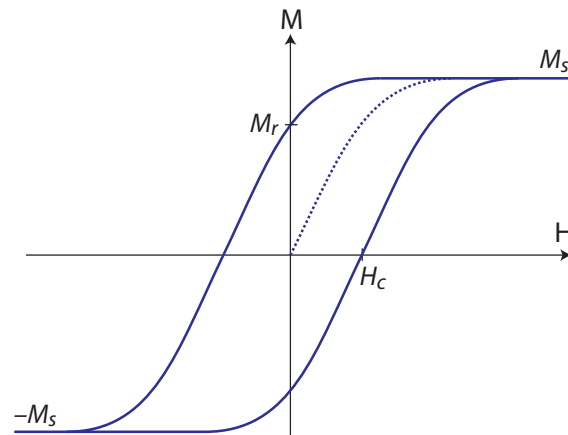
Figure 2.16: A typical hysteresis loop for a ferromagnetic material.

2.10 Hysteresis

We are now in a position to understand how magnetisation in real magnetic materials varies as a function of applied magnetising field, H. A typical example is shown in Figure 2.16. Assuming a completely de-magnetised starting point, the material is divided up into domains, as described above, with equal number/size in opposing directions to give no net magnetisation. As the applied field increases, the domains change size and rotate until the material is fully magnetised and all spins are aligned, giving the *saturation magnetisation M_s*. As the field is reduced, the energy barrier in forming domain walls, and importantly, the restricted motion of domain walls past lattice defects/impurities known as *domain wall pinning*, is such that the magnetisation doesn't follow the same curve back to zero, but rather remains after the field has been removed. The magnetisation present after the field is reduced to zero is called the *remnant magnetisation M_r*, which is often quoted in terms of a magnetic field $B_r = \mu_0 M_r$, known as the *remnant field*. When a sufficiently large reverse field is applied, the magnetisation passes through zero, before eventually reaching the reverse saturation magnetisation, and so on. The value of magnetising field required to send the magnetisation to zero is known as the *coercivity*, H_c (which can also be quoted as a magnetic field $B_c = \mu_0 H_c$.)

Through these parameters, H_c, B_r, M_s etc. we can classify a general magnetic material. In addition, the energy stored in the hysteresis curve is an important material parameter. In general this is of order $H_c B_r$, but often a bit less depending on the shape of the hysteresis loop. The product $(BH)_{\max}$ is the figure of merit which describes the maximum magnetic energy

stored per unit volume.

We can also describe in general whether materials are difficult or easy to magnetise (or, in other words, do they have wide or narrow hysteresis loops), by calling them *magnetically hard* or *magnetically soft*. Note that this has nothing to do with their hard or soft mechanical properties. In order to achieve high H_c, or make a good hard magnetic material, it is important to suppress domain wall motion through for example the introduction of crystal defects.

It is important to note that the behaviour of hysteresis loops (including their vital statistics: H_c, B_r etc.) are functions of temperature, orientation of field with respect to crystal axes, and frequency of applied field (i.e. for AC magnetic fields).

2.11   The Stoner-Wohlfarth model

The ultimate limit in restricting domain wall motion is to use a polycrystalline material where each grain is too small (e.g. < 100 nm) to contain more than one magnetic domain. The behaviour of such materials is described by the *Stoner-Wohlfarth model* which looks at the behaviour of each single-domain grain of material, and how its magnetisation rotates as a function of the applied field. In this single grain, all spins are aligned so the absolute value of the magnetisation is always M_s, the saturation magnetisation for the domain. In zero-applied field, this is aligned at angle 0 or π to the easy axis of the grain, though when a field is applied, it will be at some general angle θ to this axis. For generality, we also consider the field H is applied at some arbitrary angle ϕ to the easy axis (see Figure 2.17).

As the strength of the applied field is varied, the following energies are important: a) the magnetic energy of the domain in the applied field:

$$(2.36)$$

and b) the anisotropy energy: $U_\mathrm{ani} = K \sin^2 \theta$. Summing these together, the total energy is:

$$(2.37)$$

This function is plotted as E (energy) versus θ (orientation of magnetisation) for different strengths and orientations of the applied field (Figure 2.18). In the figure, H is varied between $\pm 2 \frac{K}{\mu_0 M_s}$.

For the case $\phi = 0$, the applied field is along the easy axis. The magnetisation is trapped in the local potential minimum until a large reverse field is applied, overcoming the anisotropy energy. This leads to a box-shaped hysteresis loop, as shown in Figure 2.19
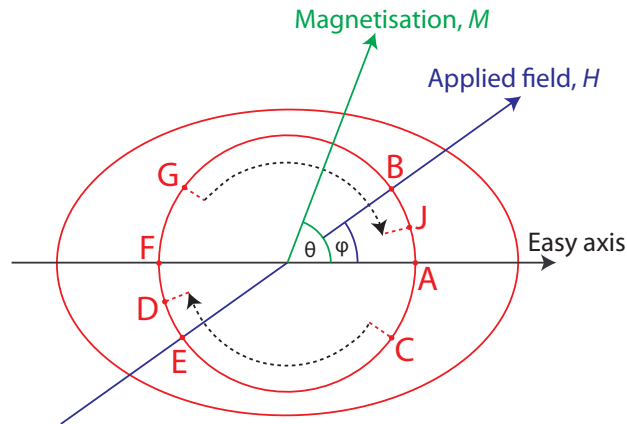
20

Figure 2.17: Magnetisation rotation in the Stoner-Wohlfarth model.

For $\phi = 30°$, the applied field is aligned slightly off the easy axis. For large applied fields, the magnetisation is aligned fully with the field orientation. As H is reduced towards zero, the magnetisation rotates until it is aligned along the easy axis for $H = 0$. It continues to rotate gradually until a critical point where the local potential minimum vanishes and the magnetisation abruptly rotates. As the field increases further, M gradually rotates further towards complete aligned with H.

Qualitatively similar behaviour is observed for larger ϕ, such as the case for $\phi = 60°$, shown. For applied field perpendicular to the easy axis ($\phi = 90°$), the hysteresis loop vanishes.
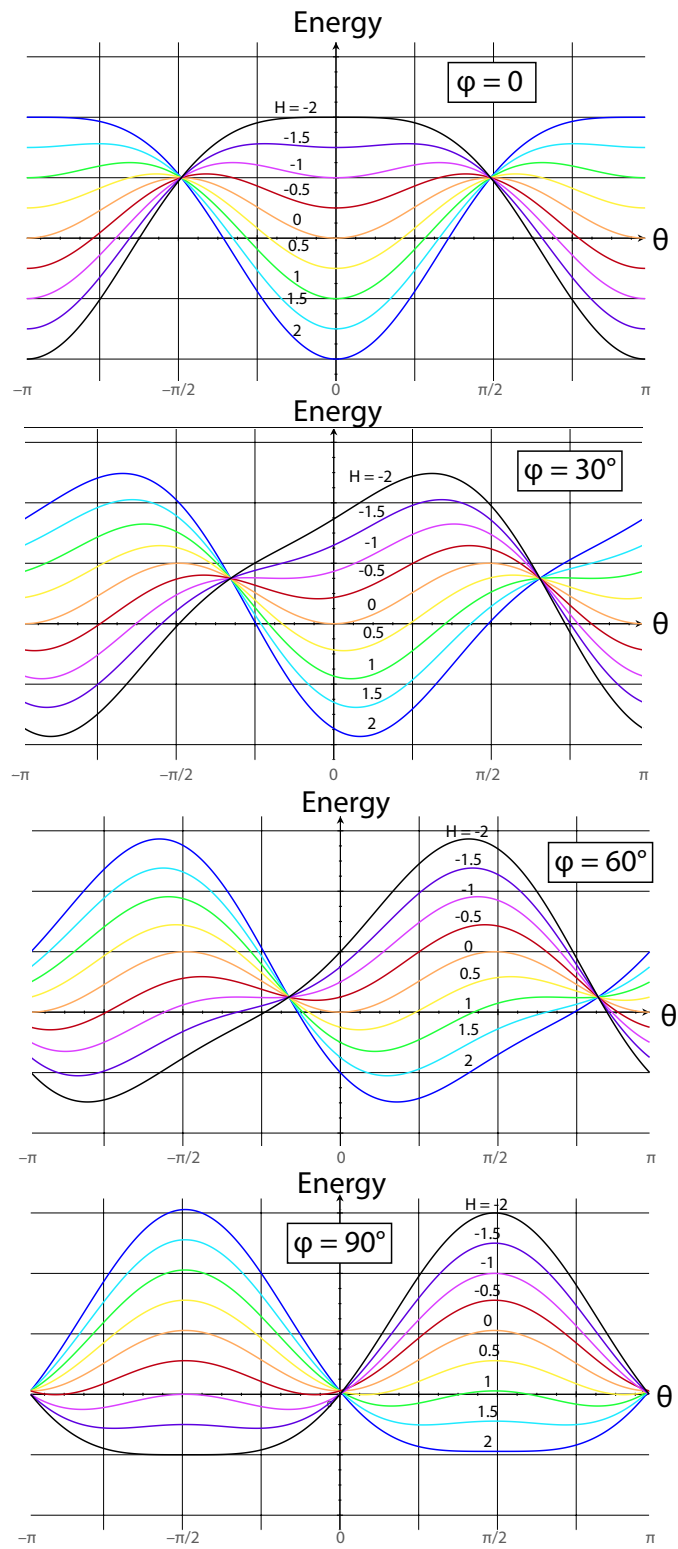
Figure 2.18: Stoner-Wohlfarth potential landscape (H has units of $K/\mu_0 M_s$).
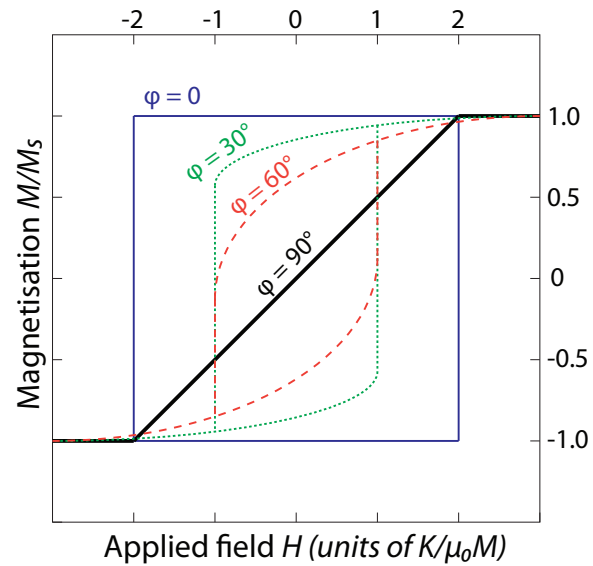
Figure 2.19: Hysteresis loop in the Stoner-Wohlfarth model, for different orientations of the applied field H to the easy axis.