B3D Handout 11: Classifying Differential Equations.

We can classify our differential equation by four properties:

- Is it an **ordinary** differential equation?
- Is it **linear**?
- Does it have **constant coefficients**?
- What is the **order**?

Ordinary

An Ordinary Differential Equation or ODE has only one independent variable (for example, x, or t), and so uses ordinary derivatives.

The alternative is a partial differential equation; we will not solve PDEs in this course.

Linearity

A differential equation is linear if every term in the equation contains none or exactly one of either the dependent variable or its derivatives. There are no products of the dependent variable with itself or its derivatives. Each term has at most one power of the equivalent of x or $\dot{x}$ or $\ddot{x}$ or $\ldots$; or $f(x)$ and its derivatives.

Examples:

$$f(x)\frac{\mathrm{d}f}{\mathrm{d}x} = -\omega^2 x$$

Nonlinear

$$\frac{\mathrm{d}f}{\mathrm{d}x} = f^3(x)$$

Nonlinear

$$\frac{\mathrm{d}^2 f}{\mathrm{d}x^2} = -x^2 f(x) + e^x$$

Linear

Constant coefficients

A differential equation has constant coefficients if the dependent variable and all its derivatives are only multiplied by constants.

Examples: do these have constant coefficients?

$$3\frac{\mathrm{d}f}{\mathrm{d}x} = -\omega^2 x$$

Yes

$$\frac{\mathrm{d}^2 f}{\mathrm{d}x^2} = -x^2 f(x) + e^x$$

No

$$\frac{\mathrm{d}^2 f}{\mathrm{d}x^2} + 3\frac{\mathrm{d}f}{\mathrm{d}x} + 2f(x) = \sin x e^x$$

Yes

Order

The order of a differential equation is the largest number of derivatives (of the dependent variable) ever taken.

Examples:

$$f(x)\frac{\mathrm{d}f}{\mathrm{d}x} = -\omega^2 x$$

First order

$$\frac{\mathrm{d}^2 f}{\mathrm{d}x^2} = -x^2 f(x) + e^x$$

Second order

$$\frac{\mathrm{d}^2 f}{\mathrm{d}x^2} + 3\frac{\mathrm{d}^2 f}{\mathrm{d}x^2}\frac{\mathrm{d}f}{\mathrm{d}x} = 0$$

Second order