B3D
Example Sheet 1.

Handed out Tuesday 10 January 2006.
Due in before the lecture on Monday 16 January 2006.

1. Consider the functions $f(x, y) = y^2(1-x)^{1/2} + x\sin(xy)$ and $u(x, y) = \dfrac{xy}{(x+y)}$.

   Recall the notations:

$$f_x = \frac{\partial f}{\partial x}; \qquad f_y = \frac{\partial f}{\partial y}; \qquad f_{xy} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right), \qquad \text{and so on.}$$

   (a) Calculate f_x and f_y.
   (b) Calculate f_{xx}, f_{yx}, f_{xy} and f_{yy} and verify that $f_{xy} = f_{yx}$.
   (c) Show that $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = 0$.

   [Hint: in part (c), simplify your expressions as far as possible at every stage.]

2. Calculate all the first and second partial derivatives of the function

$$f(x, y) = xy^2 + \sin y + e^x + 4x^2y^2.$$

3. You are standing at the point where $x = y = 100$m on a hillside whose height (in metres above sea level) is given by

$$z = 100 + \frac{1}{100}\left(x^2 - 3xy + 2y^2\right)$$

   with the positive x-axis pointing East and the positive y-axis pointing North. If you walk due East, will you initially be ascending or descending? At what angle from the horizontal?

4. Let f be a function of x and y with $x = st$ and $y = (s+t)/(s-t)$, and let

$$F(s, t) = f(x(s, t), y(s, t)).$$

   (a) Use the extended chain rule to express F_s and F_t in terms of f_x, f_y, s and t.
   (b) Show that $2xf_x = sF_s + tF_t$.
   (c) Show that $2yf_y = \frac{1}{2}(s^2 - t^2)(\frac{1}{s}F_t - \frac{1}{t}F_s)$.