

B3D

Example Sheet 10.

Not for credit.

1. Find the eigenvalues and eigenvectors of the matrix

$$\underline{\underline{A}} = \begin{pmatrix} 4 & 3 \\ -1 & 0 \end{pmatrix}.$$

Give a matrix $\underline{\underline{V}}$ for which $\underline{\underline{A}} = \underline{\underline{V}}\underline{\underline{\Lambda}}\underline{\underline{V}}^{-1}$ for $\underline{\underline{\Lambda}}$ a diagonal matrix.

Find the inverse matrix $\underline{\underline{V}}^{-1}$ and calculate directly the two products

- $\underline{\underline{V}}^{-1}\underline{\underline{A}}\underline{\underline{V}}$, which should be the diagonal matrix $\underline{\underline{\Lambda}}$, and
 - $\underline{\underline{V}}\underline{\underline{\Lambda}}\underline{\underline{V}}^{-1}$, verifying that you regain the original matrix $\underline{\underline{A}}$.
2. For each of the following systems, reduce the augmented matrix to echelon form and hence give
- the dimension N (the number of variables to solve for)
 - the rank R of the matrix and the rank of the augmented matrix
 - the most general solution.

$$(a) \quad \left(\begin{array}{ccc|c} 1 & 1 & 3 & 4 \\ -1 & 1 & -1 & -1 \\ 2 & 0 & 4 & 9 \end{array} \right)$$

$$(b) \quad \left(\begin{array}{cccc|c} 2 & 1 & 4 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 1 & 1 & 4 & 2 & 0 \\ 0 & 1 & 2 & 1 & 0 \end{array} \right)$$

$$(c) \quad \left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 14 \\ 1 & 4 & 9 & 36 \end{array} \right)$$

$$(d) \quad \left(\begin{array}{cccc|c} 2 & 4 & 1 & -1 & 2 \\ 1 & 2 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 & 4 \end{array} \right)$$