B3D

Example Sheet 2.

Handed out Monday 16 January 2006.
Q1-5 due in before the lecture on Monday 23 January 2006.

1. Let f be a function of x and y with $x = r\cos\theta$ and $y = r\sin\theta$, and let

$$F(r,\theta) = f(x(r,\theta), y(r,\theta)).$$

   (a) Use the extended chain rule to find expressions for F_r and F_θ in terms of f_x and f_y.

   (b) Combine these to get expressions for f_x and f_y in terms of F_r and F_θ.

2. Express in polar coordinates the following functions:

   (a) In plane polar coordinates:

$$f(x,y) = (x^2 - y^2)^2 + 4x^2y^2$$

   (b) In cylindrical polar coordinates:

$$g(x,y,z) = x^2 + y^2 + 2y/x + \tan z$$

   (c) In spherical polar coordinates:

$$h(x,y,z) = z/(x^2 + y^2 + z^2) + x^2 + y^2.$$

3. For a coordinate transformation $(u,v) = (u(x,y), v(x,y), w(x,y))$ the *Jacobian matrix* is:

$$\frac{\partial(u,v)}{\partial(x,y)} = \begin{pmatrix} \partial u/\partial x & \partial v/\partial x \\ \partial u/\partial y & \partial v/\partial y \end{pmatrix}$$

   (a) Find the Jacobian matrix $\partial(x,y)/\partial(r,\theta)$ of the transformation into plane polar coordinates $x = r\cos\theta$, $y = r\sin\theta$.

   (b) Calculate the determinant of the matrix $\partial(x,y)/\partial(r,\theta)$.

4. Consider the function $f(x,y) = x^3 - x + xy^2$

   (a) Find f_x and f_y.

   (b) Find the second partial derivatives f_{xx}, f_{xy} and f_{yy}.

   (c) Find and classify the stationary points of f

5. Consider the function

$$f(x,y) = x^4 - x^2 + y^2.$$

   Find and classify its stationary points.

6*. You are given the first partial derivatives of a function $f(x,y)$. Find the most general possible form of the function.

   (a) Find $f(x,y)$ if

$$\frac{\partial f}{\partial x} = 0 \qquad \frac{\partial f}{\partial y} = 3y^2.$$

   (b) Find $f(x,y)$ if

$$\frac{\partial f}{\partial x} = y - \sin x \qquad \frac{\partial f}{\partial y} = x + 4y^3.$$