

B3D

Example Sheet 5.

Handed out Monday 6 February 2006.

Due in *after reading week* before the lecture on Monday 20 February 2006.

1. For each of the following differential equations, determine the following:

- Is it linear?
- (For linear equations only:) Does it have constant coefficients?
- What order is it?

(a) $\frac{d^3y}{dx^3} + x^2 \frac{d^2y}{dx^2} + 3y = 0$

(d) $y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = \ln x$

(b) $\frac{d^2f}{dx^2} + f \frac{df}{dx} + 3 = e^x$

(e) $\frac{d^2f}{dx^2} + \sin x \frac{df}{dx} = f + 2$

(c) $\frac{d^3x}{dt^3} + 4 \frac{d^2x}{dt^2} + 3x = \tan t$

(f) $x^2 \frac{dx}{dt} + x^3 = 0.$

2. Solve the following differential equations, with the initial conditions given:

(a) $\frac{dy}{dx} + 3y = 0$, with $y(0) = 1$;

(c) $x \frac{dy}{dx} + (x-1)y = x^2$, with $y(1) = 1 + 2e^{-1}$;

(b) $\frac{dy}{dx} + 4xy = 2x$, with $y(0) = \frac{1}{2}$;

(d) $(1+x) \frac{dy}{dx} + 3y = 0$, with $y(0) = 1.$

3. Find the general solution to the following equations:

(a) $\frac{dy}{dx} + 4y = 0$;

(b) $\frac{d^2y}{dx^2} - 5 \frac{dy}{dx} + 6y = 0$;

(c) $\frac{d^2y}{dx^2} - 6 \frac{dy}{dx} + 9y = 0$;

(d) $\frac{d^2y}{dx^2} - 4 \frac{dy}{dx} + 13y = 0$;

(e) $\frac{d^5y}{dx^5} - \frac{d^3y}{dx^3} - 2 \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} = 0.$

4. Find the general solution to the following equations:

(a) $x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} - 8y = 5x^3$;

(b) $x^2 \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + 2y = 4$;

(c) $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} - y = 3x^2.$