B3D

Example Sheet 6.

Handed out Monday 20 February 2006.
Due in before the lecture on Monday 27 February 2006.

1. Find the general solution to each of the following ordinary differential equations:

   (a) $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} + 4\dfrac{\mathrm{d}y}{\mathrm{d}x} + 4y = x^2$

   (b) $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} - 4\dfrac{\mathrm{d}y}{\mathrm{d}x} + 3y = 2e^x$

   (c) $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2\dfrac{\mathrm{d}y}{\mathrm{d}x} + 5y = \cos 2x$

   (d) $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} + y = x\cos x + \sin x$

2. Solve the following differential equations, with the initial or boundary conditions given:

   (a) $\dfrac{\mathrm{d}y}{\mathrm{d}x} + \dfrac{1}{x}y = 3\cos x$, with $y(\pi) = 0$

   (b) $x^2\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} - 2x\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2x^2$, with $y(0) = 2$ and $y(1) = 3$

   (c) $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} + \dfrac{\mathrm{d}y}{\mathrm{d}x} - 6y = 2e^{3x}$, with $y(0) = y'(0) = 0$.

3. Two functions $f(x)$ and $g(x)$ are *orthogonal* in the interval $0 \le x \le 2L$ if

$$\int_0^{2L} f(x)g(x)\,\mathrm{d}x = 0.$$

   Using the formulae:

$$\sin(a+b) = \sin a\cos b + \sin b\cos a$$
$$\cos(a+b) = \cos a\cos b - \sin a\sin b$$

   (a) Show that $\cos\left(\dfrac{\pi mx}{L}\right)$ and $\cos\left(\dfrac{\pi nx}{L}\right)$ are orthogonal in the interval $0 \le x \le 2L$ when $m \ne n$. (Here m and n are integers.)

   (b) Show that $\sin\left(\dfrac{\pi mx}{L}\right)$ and $\cos\left(\dfrac{\pi nx}{L}\right)$ are orthogonal for all integers m and n.

   (c) Find

$$\int_0^{2L} \cos\left(\frac{\pi nx}{L}\right)\cos\left(\frac{\pi nx}{L}\right)\mathrm{d}x$$

   for all integers n. Be careful to check the case $n = 0$.