B3D

Example Sheet 7.

Handed out Monday 27 February 2006.
Due in before the lecture on Monday 6 March 2006.

1. The function $F(x)$ with period $2L$ is defined by

$$F(x) = \begin{cases} 0 & \text{for} \quad -L \leq x \leq -L/4 \\ 1 + (4x/L) & \text{for} \quad -L/4 \leq x \leq 0 \\ 1 - (4x/L) & \text{for} \quad 0 \leq x \leq L/4 \\ 0 & \text{for} \quad L/4 \leq x \leq L \end{cases}$$

   (a) Sketch $F(x)$ over three periods.

   (b) Is F even or odd? Find the Fourier series (i.e. the values of a_n and b_n) for $F(x)$.

   (c) For what values of n (if any) is the Fourier coefficient a_n equal to zero?

2. The "rectified wave" $f(x)$ with period 2π is

$$f(x) = \begin{cases} \sin x & \text{for } x \text{ between } 0 \text{ and } \pi \\ 0 & \text{for } x \text{ between } \pi \text{ and } 2\pi. \end{cases}$$

   (a) Sketch $f(x)$ over three periods

   (b) Calculate the Fourier series for $f(x)$:

      (i) a_0
      (ii) a_1
      (iii) b_1
      (iv) a_n for $n \geq 2$
      (v) b_n for $n \geq 2$.

3. Consider the periodic functions $f(x)$ and $g(x)$ with period $T = 2$, defined by

$$f(x) = 2x$$

   and

$$g(x) = x^2$$

   for x between -1 and 1.

   (a) Sketch $f(x)$ and $g(x)$ for three periods

   (b) Find the Fourier series for $f(x)$. Write out the first three non-zero terms in the series.

   (c) Find a_0 for the Fourier series for $g(x)$.

   (d) Integrate $f(x)$ and its Fourier series from 0 to A (with $A < 1$): that is, integrate each term in the Fourier series to obtain a new series. Deduce a_n ($n \geq 1$) for the Fourier series for $g(x)$.

   (e) Use your results to parts (c) and (d) above to find the sum of an infinite series.