

B3D

Example Sheet 8.

Revision from B3C

Handed out Monday 6 March 2006.

Due in before the lecture on Tuesday 14 March 2006.

1. Using row reduction to echelon form, find the general solution of the linear equations:

(a)

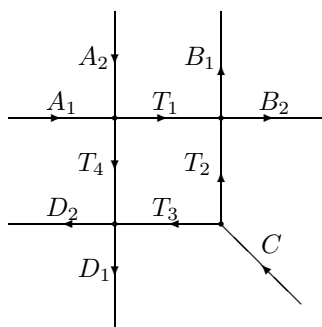
$$\begin{aligned} x - 2y + 3z &= 0 \\ x + 4y - 6z &= 0 \\ 2x + 2y - 3z &= 0 \end{aligned}$$

(b)

$$\begin{aligned} x_1 - 3x_2 - 2x_3 + 3x_5 &= 1 \\ & x_3 + 3x_4 + 5x_5 = -1 \\ x_1 - 3x_2 + x_3 + x_4 + 2x_5 &= 6 \\ & x_3 + x_4 + x_5 = 1 \end{aligned}$$

What is the rank of the augmented matrix in each case? Note that, in each case, the number of free parameters in the general solution equals the number of variables minus the rank.

2. The diagram shows a network of one-way streets. The outer traffic flow rates (in the directions indicated) are $A_1 = 200$ vehicles per hour, $A_2 = 100$, $B_1 = 150$, $B_2 = 250$, $C = 700$, $D_1 = 300$, $D_2 = 300$.



Write down the linear equations relating the inner flow rates T_1, T_2, T_3, T_4 and the outer flow rates A_1 etc., using the rule that, at each junction, traffic in = traffic out. Find a general solution for T_1, T_2, T_3, T_4 . (NB it is not unique). In the range of sensible solutions, for which the flow rates cannot be negative, what is the maximum value for each T ?

- 3.

$$\underline{\underline{A}} = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

- (a) Find the eigenvalues and eigenvectors of $\underline{\underline{A}}$.
 (b) For the multiple eigenvalue, find all possible vector solutions of the equation

$$(\underline{\underline{A}} - \lambda \underline{\underline{I}})^2 \underline{\underline{w}} = 0$$

[Hint: find $(\underline{\underline{A}} - \lambda \underline{\underline{I}})^2$ and then use Gaussian elimination.]