Summary: What tools apply and when?

Algebraic equations

1. Set $\varepsilon = 0$ and solve the resulting system. Do you have enough solutions?
2. [**YES**]: For each root, set $\delta_0 = 1$ and go to 4.
3. [**NO**]: Seek leading-order scalings $x = \delta_0 x_0$ that work with x_0 strictly order 1.
4. For each root, you now know δ_0. Try the expansion $x = \delta_0 x_0 + \varepsilon \delta_0 x_1 + \cdots$
   - Does it work?
   - [**YES**] Done!
   - [**NO**], I have an equation with no solutions.
     - Try a different scaling $x = \delta_0 x_0 + \delta_1 x_1$ and look for values of δ that make leading terms balance (after eliminating x_0 terms from the equation).

Nonlinear Differential Equations

1. Do you have boundary conditions that fix the scale of f?
2. **YES**: Fix the scale of f and allow stretches $x = a + \delta z$. Treat as linear differential equations (below).
3. **NO**: Allow both scale and stretch $f = \varepsilon^\alpha F$, $x = a + \varepsilon^\beta z$. Graph all possible balances and identify any distinguished scalings.

Linear Differential Equations

1. Identify all working stretches $x = a + \delta z$. For each stretch:
   - Write down the new equation to solve for $f(z)$
   - Try an expansion $f = f_0 + \varepsilon f_1 + \cdots$. Does it work?
     - [**YES**] Go on to 2.
     - [**NO**], I have an equation with no solutions.
       * Try a different scaling $f = f_0 + \delta f_1$ and look for values of δ that make leading terms balance (after eliminating f_0 terms from the equation).
2. You now have n distinct solutions (for an nth order ODE). Choose which one to use where. Helpful rules:
   - You will need the *outer* (unstretched) solution in most of the domain
   - An *inner* (boundary layer) solution must **decay exponentially** as you move into the outer region.
3. Apply the boundary conditions to the relevant solution at each boundary.
4. Use intermediate variable matching to determine the remaining unknowns.