Workshop 1

1. Let $L_1(t) = \langle 1 + t, 2, 3 - t \rangle$ be a line in $\mathbb{R}^3$.

a) Find the point p on the line L_1 that is closest to the origin.

b) Find the Cartesian equation of the plane $\mathcal{P}$ containing L_1 and the point $q = (1, 0, -1)$.

c) Find the equation of the line L_2 which is perpendicular to L_1, contains the point p, and is contained by the plane $\mathcal{P}$.

2. A spaceship maneuvering in space, far from any gravitational influences, is executing a predetermined acceleration program which yields a position vector $\mathbf{r}(t)$ for the ship, relative to a small space beacon, given by

$$\mathbf{r}(t) = (t-2)\mathbf{i} + (t-3)^2\mathbf{j} + (t-4)^3\mathbf{k}.$$

a) Suppose that the captain shuts down the engines at time t_0. Find the subsequent motion of the ship.

b) Show that if t_0 is chosen appropriately then the ship will hit the beacon.

3. Find equations for two orthogonal planes, both of which contain the line $\mathbf{v} = \langle 1, 0, 3 \rangle + t \langle -1, 2, 1 \rangle$, one of which passes through the origin.

4. Suppose that $\vec{v}$ is a vector in $\mathbf{R}^3$ which is not the zero vector.

a) If $\vec{v} \cdot \vec{w} = \vec{v} \cdot \vec{q}$, must it be true that $\vec{w} = \vec{q}$?

b) If $\vec{v} \times \vec{w} = \vec{v} \times \vec{q}$, must it be true that $\vec{w} = \vec{q}$?

c) If $\vec{v} \cdot \vec{w} = \vec{v} \cdot \vec{q}$ and $\vec{v} \times \vec{w} = \vec{v} \times \vec{q}$, must it be true that $\vec{w} = \vec{q}$?