Workshop 4

1. A flat circular plate has the shape of the region $x^2 + y^2 \leq 1$. The plate (including the boundary $x^2 + y^2 = 1$) is heated so that the temperature T at any point (x, y) is given by $T(x, y) = x^3 - x + y^2$. Locate the hottest and coldest points of the plate and determine the temperature at each of those points.

2. a) What is the maximum value of the function $f(x, y) = 3x + 5y$ subject to the constraint $x^2 + y^2 = 1$, and where is it attained? Draw a picture of the constraint and the appropriate level set of the objective function.

b) Suppose n is a positive real number. What is the maximum value of the function $f(x, y) = 3x + 5y$ subject to the constraint $x^n + y^n = 1$ and where is it attained? Your answers should all be functions of n.

c) What happens to the maximum value found in b) when $n \to \infty$? Try to draw a picture of the constraint and the level set when n is large.

d) What happens to the maximum value found in b) when $n \to 0^+$? Try to draw a picture of the constraint and the level set when n is small.

3. a) Suppose that $z = f(x, y)$, that $x = g(t)$ and $y = h(t)$, and that the functions f, g, and h are twice differentiable. Use the Chain Rule to find expressions for $\frac{dz}{dt}$ and $\frac{d^2z}{dt^2}$.

b) An insect crawls on a metal plate in the plane. At time $t = 1$ its position vector is $\mathbf{i} + 2\mathbf{j}$, its velocity is $2\mathbf{i} - \mathbf{j}$, and its acceleration is $3\mathbf{i} + 4\mathbf{j}$. Suppose that the temperature of the plate at the point x, y is a certain function $T(x, y)$ satisfying

$$\begin{array}{lll} T(1,2) = 2, & T_x(1,2) = -1, & T_y(1,2) = 3, \\ T_{xx}(1,2) = 0, & T_{xy}(1,2) = 1, & T_{yy}(1,2) = -2. \end{array}$$

If $T(t)$ is the temperature experienced by the insect at time t, find $\frac{dT}{dt}$ and $\frac{d^2T}{dt^2}$ at time $t = 1$.

4. A rectangular box is to be constructed with volume one cubic foot. The material used in the top and bottom costs a dollars per square foot, in the front and back b dollars per square foot, and in the sides c dollars per square foot. Find the dimensions which will produce the cheapest box.