Workshop 5

1. a) Suppose $f(x, y, z)$ is defined in a region $\mathcal{R}$ of $\mathbf{R}^3$. The average value of f over $\mathcal{R}$ is $\iiint_{\mathcal{R}} f(x, y, z)\, dv$ divided by $\iiint_{\mathcal{R}} 1\, dv$.

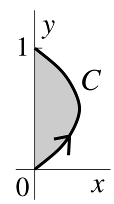
> A bat flies in and around a hemispherical cave, with water at the bottom, so it cannot land there. If the radius of the cave is R, what is the average height of the cave to the bat? (The bat flies totally at random throughout all of the space available to it.)

b) Suppose $f(x, y, z)$ is defined on a surface $\mathcal{S}$ in $\mathbf{R}^3$. The average value of f over $\mathcal{S}$ is $\iint_{\mathcal{S}} f(x, y, z)\, dS$ divided by $\iint_{\mathcal{S}} 1\, dS$.

> A *non-swimming* slug crawls about on the inner surface of the same cave as described above. Its motion is confined to that surface of the cave. What is the average height of the cave to the slug? (The slug crawls totally at random throughout all of the space available to it.)

c) Which creature is higher (on average)?

2. Suppose C is the part of the parabola $x = -y(y - 1)$ connecting $(0, 0)$ to $(0, 1)$ as shown.



Compute

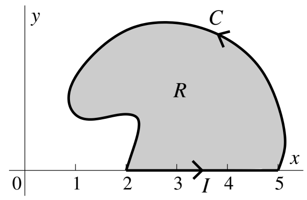
$$\int_C \left(e^{(x^2)} + 5y + 3\right) dx + \left(x^2 + 5y^3\right) dy\,.$$

Hint Use Green's Theorem.

3. A region R in $\mathbf{R}^2$ is located in the first quadrant, as shown. Its boundary, oriented counterclockwise as shown, is an interval $I = [2, 5]$ on the x-axis and a curve C in the first quadrant.

Suppose the following information is also known:

$$\iint_R 1\, dA = 5\,; \quad \iint_R x\, dA = 12\,; \quad \iint_R y\, dA = 8\,.$$



Find $\displaystyle\int_C \left(x^2 + xy + 3y\right) dx + \left(\arctan(y^3) + 3x^2 + 2xy + x\right) dy.$