FREE ELECTRON MODEL

Lecture 16

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6.4.4 Experimental results

For our typical metal, with $n \approx 6 \times 10^{28}\mathrm{m}^{-3}$ **and** $\sigma \approx 6 \times 10^{7}\,\Omega^{-1}\mathrm{m}^{-1}$ **this gives** $\tau \approx 3 \times 10^{-14}$ s**. Putting this together with the Fermi velocity** $v_{\mathrm{F}} \approx 10^{6}$ m s^{-1} **gives** $\Lambda \approx 3 \times 10^{-8}$ m **or about 100 interatomic distances.** ***Historical note:*** **Drude's theory of metals used a** ***classical*** **free electron model. This had electron speeds which were classically thermal (**$\frac{1}{2}mv^2 = \frac{3}{2}k_{\mathrm{B}}T$**), i.e. much slower than the** v_{F} **of the Fermi gas. Drude also assumed that the electrons would be scattered by every atom (i.e. his** Λ **was about 100 times too small). As a result of these cancelling errors, his estimate of the electrical conductivity was not too bad.**

6.5 Electronic Thermal Conductivity

We can use exactly the same expression as we used for phonons:

$$\kappa = \frac{1}{3} c_V v \Lambda,$$

only now c_V **is the electronic specific heat per unit volume,** v **is the electron velocity, which we take as the Fermi velocity** v_F**, and** Λ **is the electronic mean free path,** $\Lambda = v_\mathrm{F}\tau$. **We know that**

$$c_V = \frac{\pi^2 n {k_\mathrm{B}}^2 T}{2E_\mathrm{F}},$$

(Note that we have converted from N_e **to** $n = N_e/V$ **to get specific heat per volume) and so**

$$\kappa = \frac{1}{3} \frac{\pi^2 n {k_\mathrm{B}}^2 T}{2E_\mathrm{F}} v_\mathrm{F} \times v_\mathrm{F}\tau.$$

$$\kappa = \frac{1}{3}\frac{\pi^2 n k_B{}^2 T}{2E_F} v_F \times v_F \tau,$$

with

$$E_F = \frac{1}{2} m v_F{}^2,$$

gives

$$\kappa = \frac{\pi^2 n k_B{}^2 T \tau}{3m}.$$

If we take $n = 6 \times 10^{28}\ \mathrm{m}^{-3}$ **and** $\tau = 3 \times 10^{-14}\ \mathrm{s}$ **we have at 300 K that** $\kappa = 370\ \mathrm{W\ m^{-1}K^{-1}}$**. The measured thermal conductivity of Copper is** $400\ \mathrm{W\ m^{-1}K^{-1}}$ **In pure metals, most of the thermal conductivity arises from the electrons: in impure metals (or random alloys, which amounts to the same thing) the vibrational contribution can be similar.**

6.5.1 The Wiedemann-Franz law

At metals at temperatures that are not very low, the ratio of thermal to electrical conductivity is directly proportional to temperature.

$$\frac{\kappa}{\sigma} = \frac{\pi^2 n k_\mathrm{B}{}^2 T\tau/3m}{ne^2\tau/m} = \frac{\pi^2}{3}\left(\frac{k_\mathrm{B}}{e}\right)^2 T.$$

The constant of proportionality is called the *Lorenz number*:

$$L = \frac{\kappa}{\sigma T} = \frac{\pi^2}{3}\left(\frac{k_\mathrm{B}}{e}\right)^2 = 2.45 \times 10^{-8}\,\mathrm{W}\,\Omega\,\mathrm{K}^{-2}.$$

Experimental Lorenz numbers:

$L \times 10^8 W\ \Omega\ K^{-2}$			$L \times 10^8 W\ \Omega\ K^{-2}$		
Element	L **at 273 K**	L **at 373 K**	**Element**	L **at 273 K**	L **at 373 K**
Ag	**2.31**	**2.37**	**Pb**	**2.47**	**2.56**
Au	**2.35**	**2.40**	**Pt**	**2.51**	**2.60**
Cd	**2.42**	**2.43**	**Sn**	**2.52**	**2.40**
Cu	**2.23**	**2.33**	**W**	**3.04**	**3.20**
Mo	**2.61**	**2.79**	**Zn**	**2.31**	**2.33**

A temperature-independent Lorenz number depends on the relaxation processes for electrical and thermal conductivity being the same – which is not true at all temperatures.

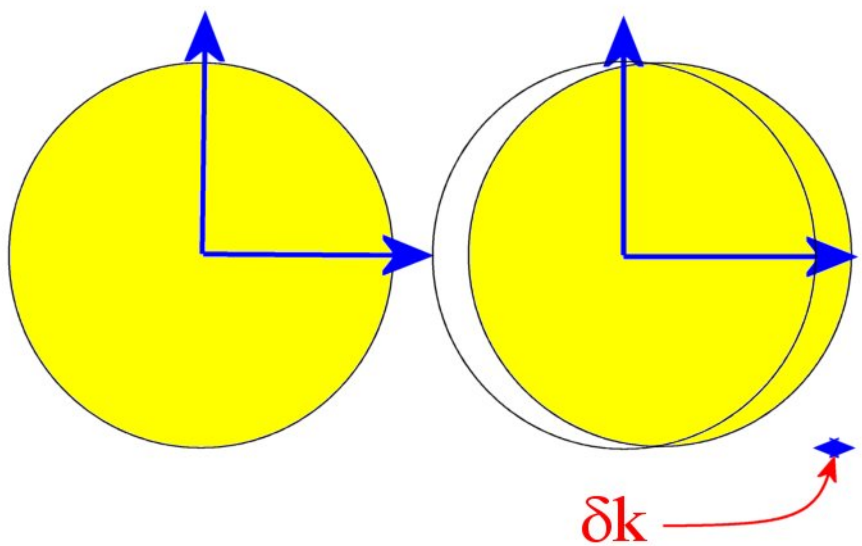
6.6 Conductivity – the view from reciprocal space

6.6.1 Electrical conductivity

The effect of a force $\mathcal{F}$ is to alter the *momentum*, $\hbar\mathrm{k}$. We can ask what this will do to the Fermi sphere. For every electron

$$\frac{\mathrm{d}\mathbf{k}}{\mathrm{d}t} = \frac{\mathcal{F}}{\hbar},$$

so the Fermi sphere is displaced sideways.



Note that there is a nett flow of electrons.

If the field acts for a time τ

$$\delta k = \mathbf{k}(\tau) - \mathbf{k}(0) = \frac{\mathcal{F}\tau}{\hbar} - \frac{e\mathcal{E}\tau}{\hbar},$$

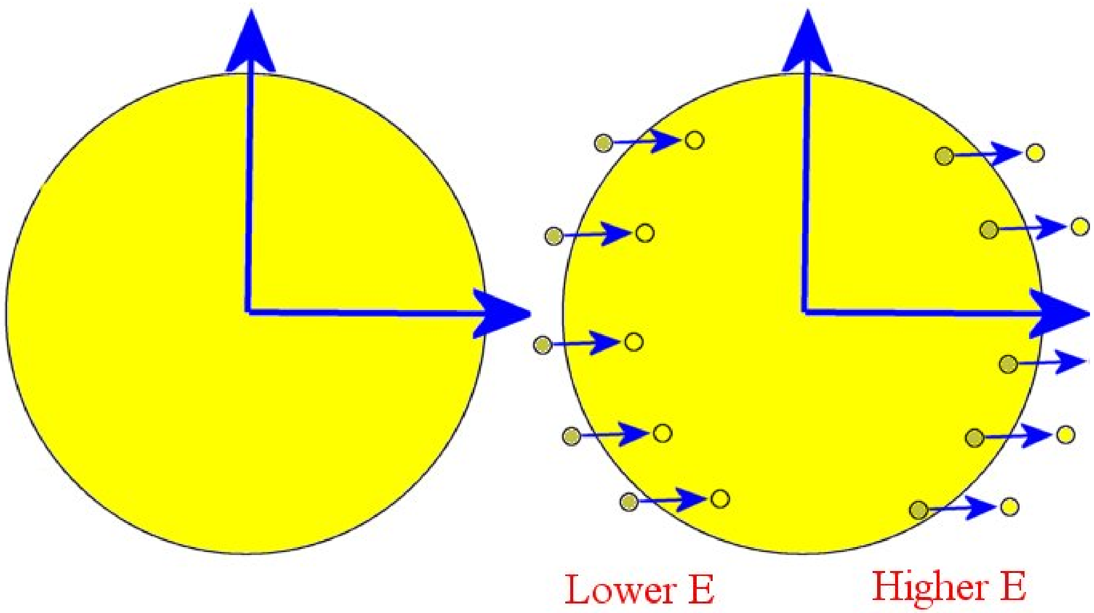
If $\mathcal{E} = 1000 \text{ V m}^{-1}$ and $\tau = 10^{-14}$ s then

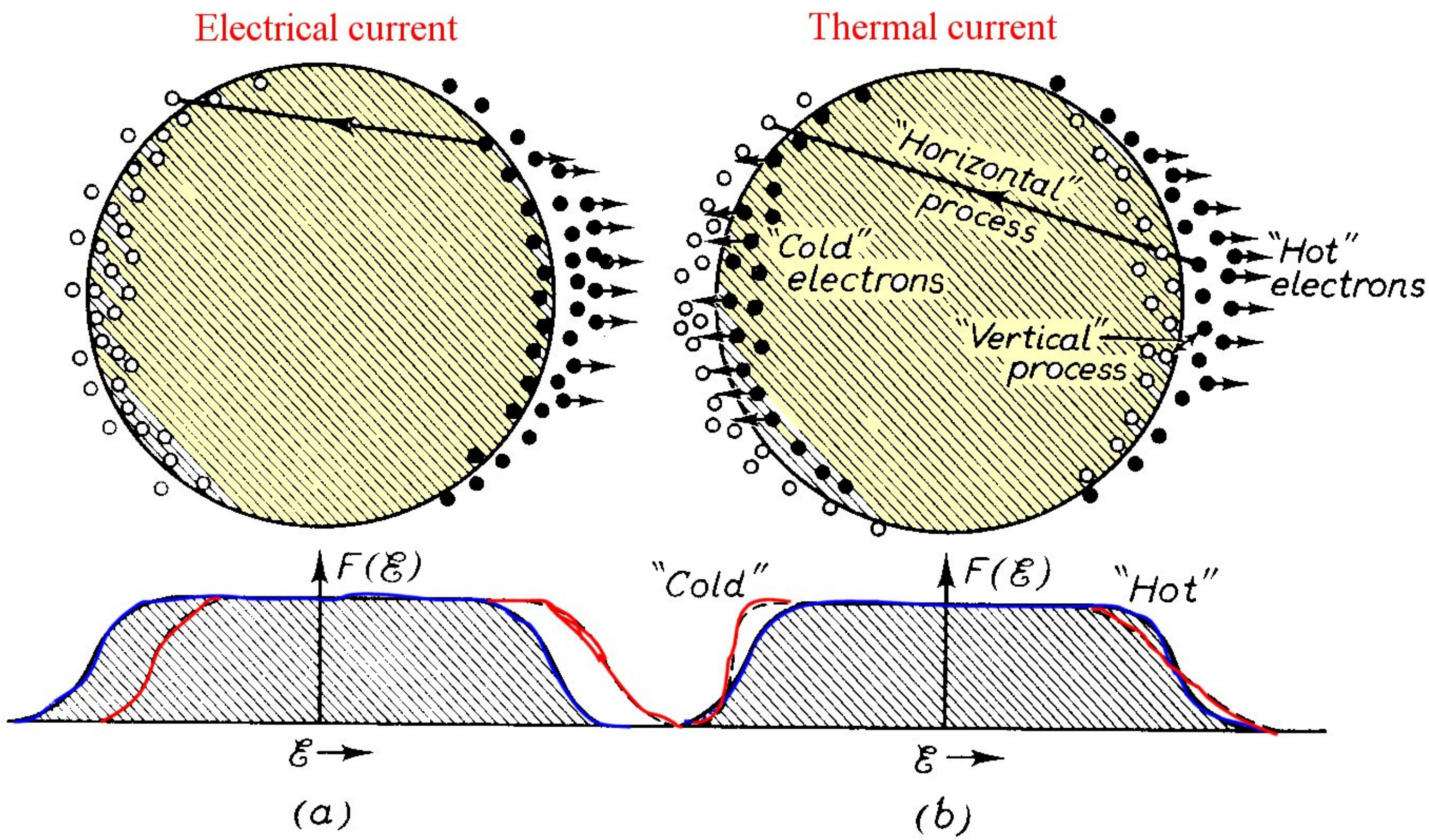
$$\delta k = \frac{1.6 \times 10^{-19} \times 1000 \times 10^{-14}}{1.05 \times 10^{-34}} \approx 10^4 \text{ m}^{-1} \approx 10^{-6}\, k_{\mathrm{F}},$$

the alteration in the Fermi surface is small.

6.6.2 Thermal conductivity

There is no nett electric current – but electrons travelling in one direction have on average higher energy than those travelling in the opposite direction.





The scattering processes are different:

- **to reduce electric current requires large change in wavevector – phonon contribution falls off quickly at low T.**
- **to reduce thermal current requires change in thermal energy – by definition, energy $\approx k_{\mathrm{B}}T$**

6.6.3 Contributions to scattering

Impurities **contribution independent of temperature**

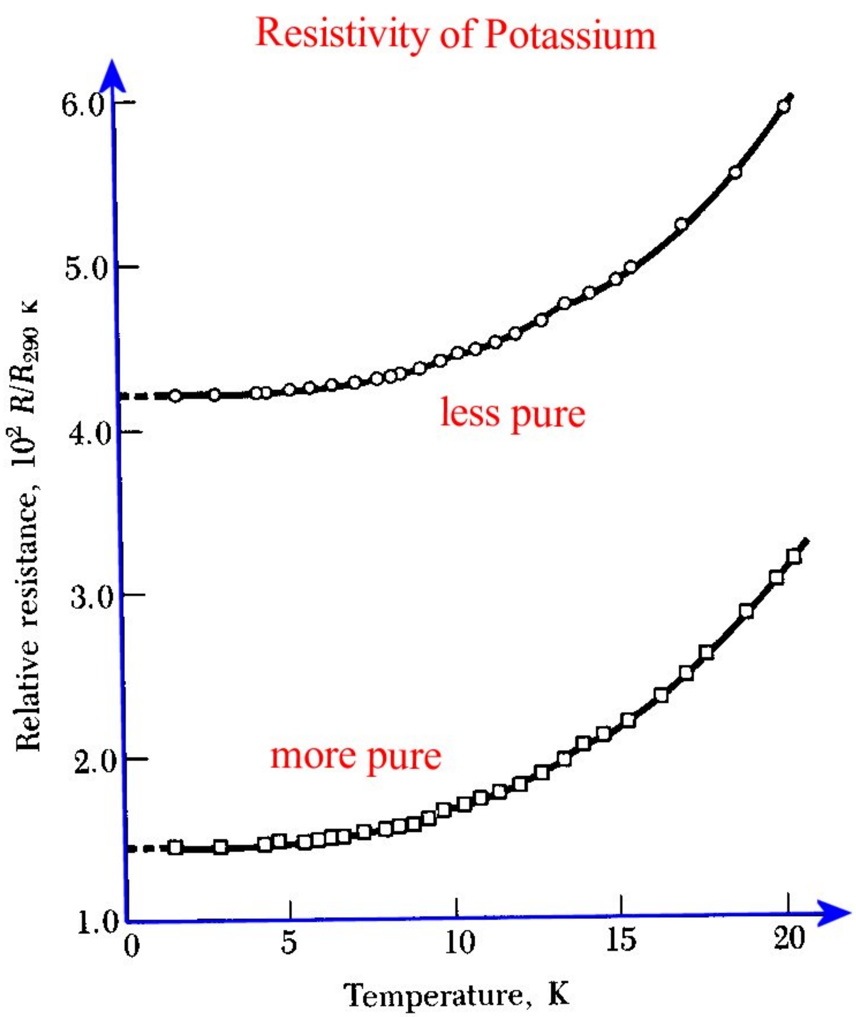
Electron-phonon scattering

- **High** T**: plenty of large-**k **phonons, so effect on** σ **and** κ **similar**
  - **number of phonons** $\propto T$ **so** $\Lambda \propto 1/T$
  - $c_{\mathrm{V}} \propto T$
  - $\sigma \propto 1/T$**,** κ **independent of** T
- **Low** T**: few large-**k **phonons, so phonons less effective at limiting** σ **than** κ
  - **number of phonons** $\propto T^3$ **so** $\Lambda \propto 1/T^3$ **for** κ
  - **number of large-**k **phonons** $\propto \exp(-\theta/T)$ **so** $\Lambda \propto \exp(\theta/T)$ **for** σ
  - $c_{\mathrm{V}} \propto T$
  - $\sigma \propto \exp(\theta/T)$**,** $\kappa \propto T^{-2}$
- **Very low** T**: very few phonons, so impurities dominate**
  - $c_{\mathrm{V}} \propto T$
  - σ **independent of** T**,** $\kappa \propto T$

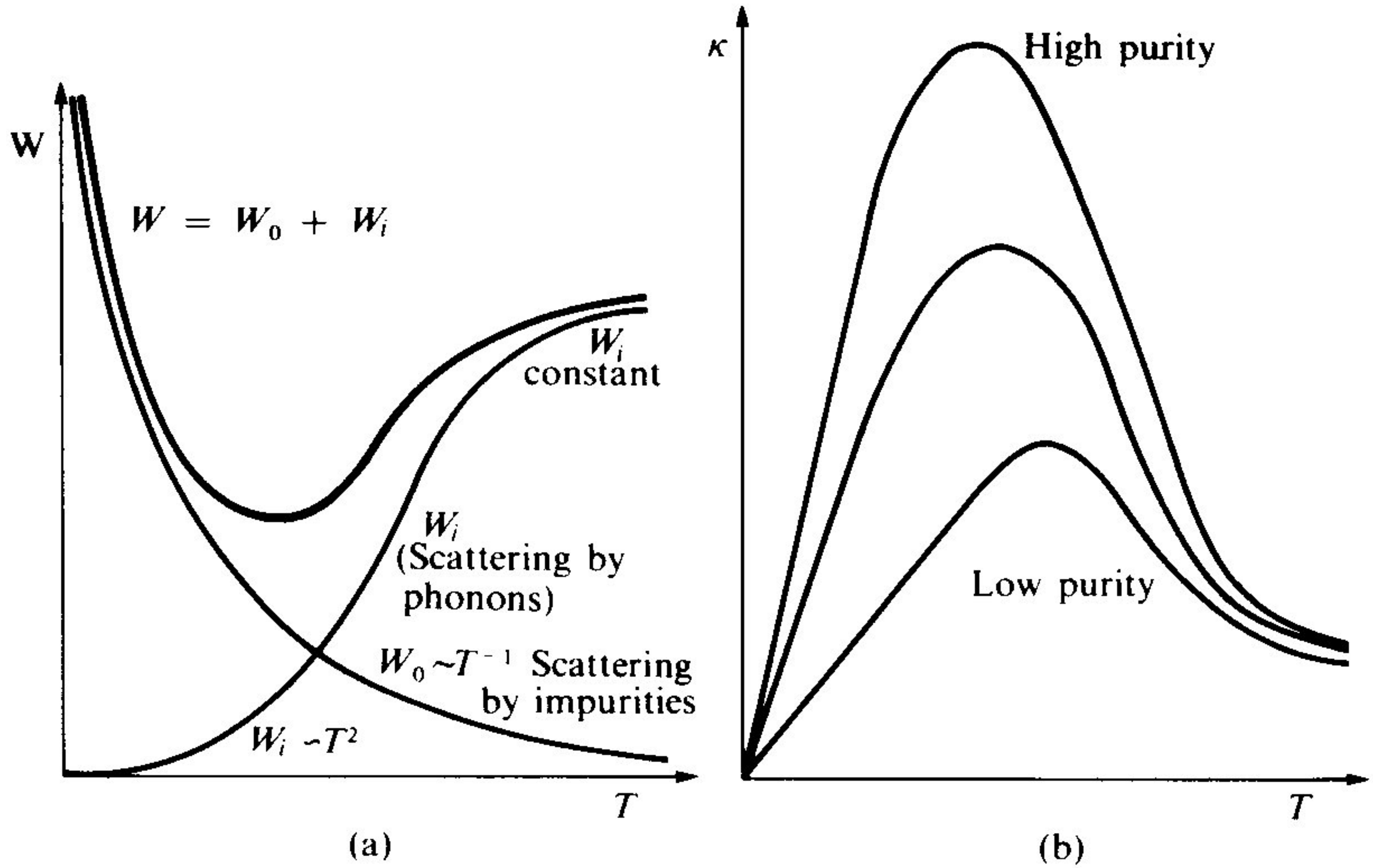
As we saw before, different processes give resistances in series:

$$\rho = \sum_i \rho_i.$$

Resistivity of potassium - different purities.



Schematic variation of thermal resistance (a), thermal conductivity (b) with T at low T.



Note that this means the Lorenz number $L = \kappa/(\sigma T)$ is not constant with temperature.