

## FREE ELECTRON MODEL

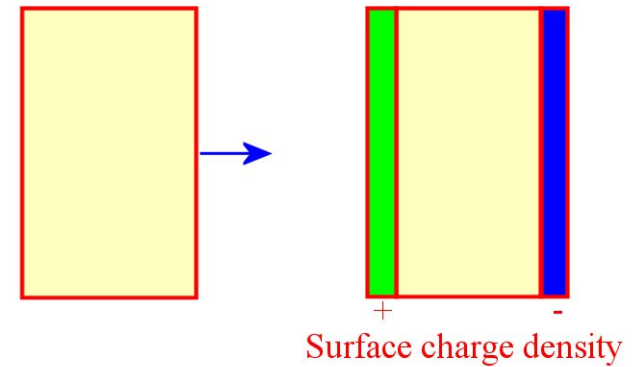
### Lecture 17

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## Magnetic Effects

### 6.7 Plasma Oscillations

The picture of a free electron gas and a positive charge background offers the possibility of *plasma oscillations* – a collective motion of all the electrons relative to the background.



If electron gas,  $n$  electrons per volume, moves a distance  $x$  relative to the positive background this gives a surface charge density

$$\sigma = -enx$$

on the positive  $x$  side. But this gives an electric field

$$\mathcal{E} = -\frac{\sigma}{\epsilon_0},$$

which tries to restore the electrons to their equilibrium position by exerting a force

$$\mathcal{F} = -e\mathcal{E} = -\frac{ne^2}{\epsilon_0}x$$

on each electron. So

$$m\ddot{x} = -\frac{ne^2}{\epsilon_0}x,$$

which is a simple harmonic oscillator with angular frequency  $\omega_P$

$$\omega_P^2 = \frac{ne^2}{\epsilon_0 m}$$

For example, if  $n = 6 \times 10^{28} \text{ m}^{-3}$ ,

$$\omega_P = \sqrt{\frac{ne^2}{\epsilon_0 m}} = \sqrt{\frac{6 \times 10^{28} \times (1.6 \times 10^{-19})^2}{8.854 \times 10^{-12} \times 9.11 \times 10^{-31}}} = 1.4 \times 10^{16} \text{ rad s}^{-1}.$$

This corresponds to an energy

$$\hbar\omega_P = 8.9 \text{ eV}.$$

If high energy (1 to 10 keV) electrons are fired through a metal film, they can lose energy by exciting plasma oscillations, or *plasmons*.

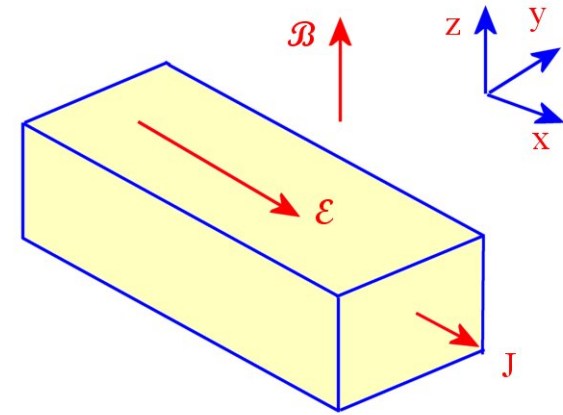
Volume plasmon energies, eV

Metal	Measured	Calculated
Li	7.12	8.02
Na	5.71	5.95
K	3.72	4.29
Mg	10.6	10.9
Al	15.3	15.8

Another success for free electron theory.

## 6.8 The Hall Effect

In a Hall experiment a magnetic field applied perpendicular to an electric current flowing along a bar.



We need to extend our previous equation by including the Lorentz force  $q\mathbf{v} \times \mathbf{B}$ .

Signs always cause problems in the Hall effect: avoid some confusion by writing  $q$  for the charge on the particles carrying the current –  $q$  includes the sign. The new transport equation is

$$m \left( \frac{d\mathbf{v}_d}{dt} + \frac{\mathbf{v}_d}{\tau} \right) = q(\mathcal{E} + \mathbf{v}_d \times \mathbf{B}).$$

Assume that  $\mathbf{B} = (0, 0, B_z)$  and  $\mathcal{E} = (\mathcal{E}_x, \mathcal{E}_y, \mathcal{E}_z)$  so

$$m \frac{dv_{dx}}{dt} + m \frac{v_{dx}}{\tau} = q\mathcal{E}_x + qv_{dy}B_z,$$

$$m \frac{dv_{dy}}{dt} + m \frac{v_{dy}}{\tau} = q\mathcal{E}_y - qv_{dx}B_z,$$

$$m \frac{dv_{dz}}{dt} + m \frac{v_{dz}}{\tau} = q\mathcal{E}_z.$$

Now we know that current can only flow in the  $x$  direction, so  $v_{dy} = v_{dz} = 0$ , and so in a steady state

$$\begin{aligned} m \frac{v_{dx}}{\tau} &= q\mathcal{E}_x, \\ 0 &= q\mathcal{E}_y - qv_{dx}\mathcal{B}_z, \\ 0 &= q\mathcal{E}_z. \end{aligned}$$

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The first equation is one we have seen before:

$$v_{dx} = \frac{q\tau}{m}\mathcal{E}_x,$$

giving the current along the bar.

The third equation states that there is no electric field in the  $z$  direction.

The second equation states that an electric field is set up in the  $y$  direction:

$$\mathcal{E}_y = v_{dx}\mathcal{B}_z.$$

Physically what happens is that the charges are accelerated in the  $y$  direction by the magnetic field, and pile up on the edges of the bar until they produce enough of an electric field to oppose the effect of the magnetic field.

We know that the current density  $J_x$  in the  $x$  direction is

$$J_x = nqv_{dx},$$

so

$$\mathcal{E}_y = \frac{J_x\mathcal{B}_z}{nq},$$

and we define the Hall coefficient as

$$R_H = \frac{\mathcal{E}_y}{J_x\mathcal{B}_z}.$$

For a free electron metal with  $n$  electrons per volume, then,  $R_H$  is negative,

$$R_H = -\frac{1}{ne}.$$

Note that measuring Hall effects in metals is *difficult*: even with high current density ( $10^6 \text{ Am}^{-2}$  and magnetic fields of order 1 T we have to measure fields

$$\mathcal{E}_y = \frac{10^6 \times 1}{6 \times 10^{28} \times 1.6 \times 10^{-19}} = 0.0001 \text{ V m}^{-1},$$

or a potential difference of less than  $1 \mu\text{V}$  on a typically-sized sample.

Metal	Valence	$R_H^{\text{theor}}/R_H^{\text{exp}}$
Li	1	0.8
Na	1	1.2
K	1	1.1
Rb	1	1.0
Cs	1	0.9

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Li	1	0.8
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K	1	1.1
Rb	1	1.0
Cs	1	0.9
Cu	1	1.5
Ag	1	1.3
Au	1	1.5
Be	2	-0.2
Cd	2	-1.2
Zn	3	-0.8
Al	3	-0.3

Alkali metals OK, noble metals numerically incorrect, higher-valent metals *wrong sign*. Major problem for free-electron theory. In addition,  $R_H$  depends on  $\mathcal{B}$  and  $T$ .

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## 6.9 Free electron approximation – final comments

We have still not explained how we can justify the assumption that electrons, charged particles, do not interact with one another. There are two effects – electrostatic screening and the exclusion principle.

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### 6.9.1 Screening

If the electrons are free to move, they arrange themselves so as to make the metal locally neutral – but if they try to pack together more densely this will increase their energy because  $E_F$ , the energy relative to the local potential, increases with  $n = N_e/V$ . As a result, the electrostatic potential round a point charge  $q$  in a free electron gas is not

$$\mathcal{V}_0(r) = \frac{q}{4\pi\epsilon_0 r},$$

but

$$\mathcal{V}(r) = \frac{qe^{-r/\lambda}}{4\pi\epsilon_0 r},$$

a *screened Coulomb potential*, with

$$\lambda = \sqrt{\frac{2\epsilon_0 E_F}{3e^2 n}} \approx 6 \times 10^{-11} \text{ m}$$

for our usual set of parameters, so that electric fields inside a metal are screened out within a few interatomic spacings.

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### 6.9.2 Electron-electron scattering

At absolute zero, scattering *cannot* occur, because of the exclusion principle:

The two electrons are initially both in occupied states inside the Fermi surface.

To conserve energy and momentum, either both final states lie inside the Fermi surface – but those states are all occupied – or one lies outside – but then the other lies inside.

Scattering is forbidden at  $T = 0$ .

At finite  $T$  there is a layer of partly occupied states near  $E_F$ , amounting to a fraction about  $k_B T/E_F$  of the electrons, giving weak scattering with probability  $\propto T^2$ .

See contribution to electrical resistivity  $\propto T^2$  in very pure metals at very low  $T$ .

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### 6.9.3 Binding energy of metals

The terms in the energy are:

- Electronic kinetic energy (reduced by allowing them to be delocalised)
- Attraction of electrons to ion cores (less than in free atoms as electrons are further from nuclei)
- Mutual repulsion of ion cores (screened by the free electron gas)
- Electron-electron repulsion (reduced by spreading out electrons)
- Quantum mechanical exchange potential between electrons
- Correlation energy (beyond single-electron wave-functions)

Balance of effects – typically a few eV per atom.

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