

Solid State Physics

SEMICONDUCTORS - III

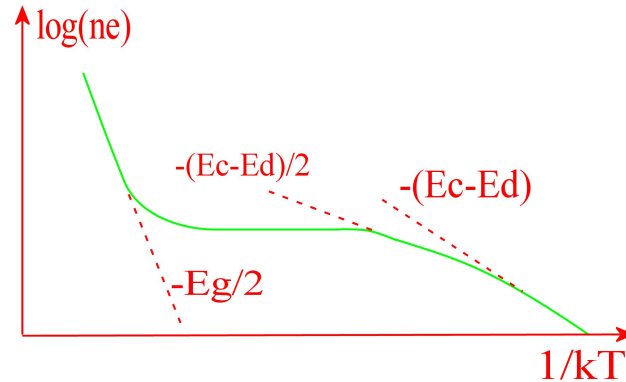
Lecture 24

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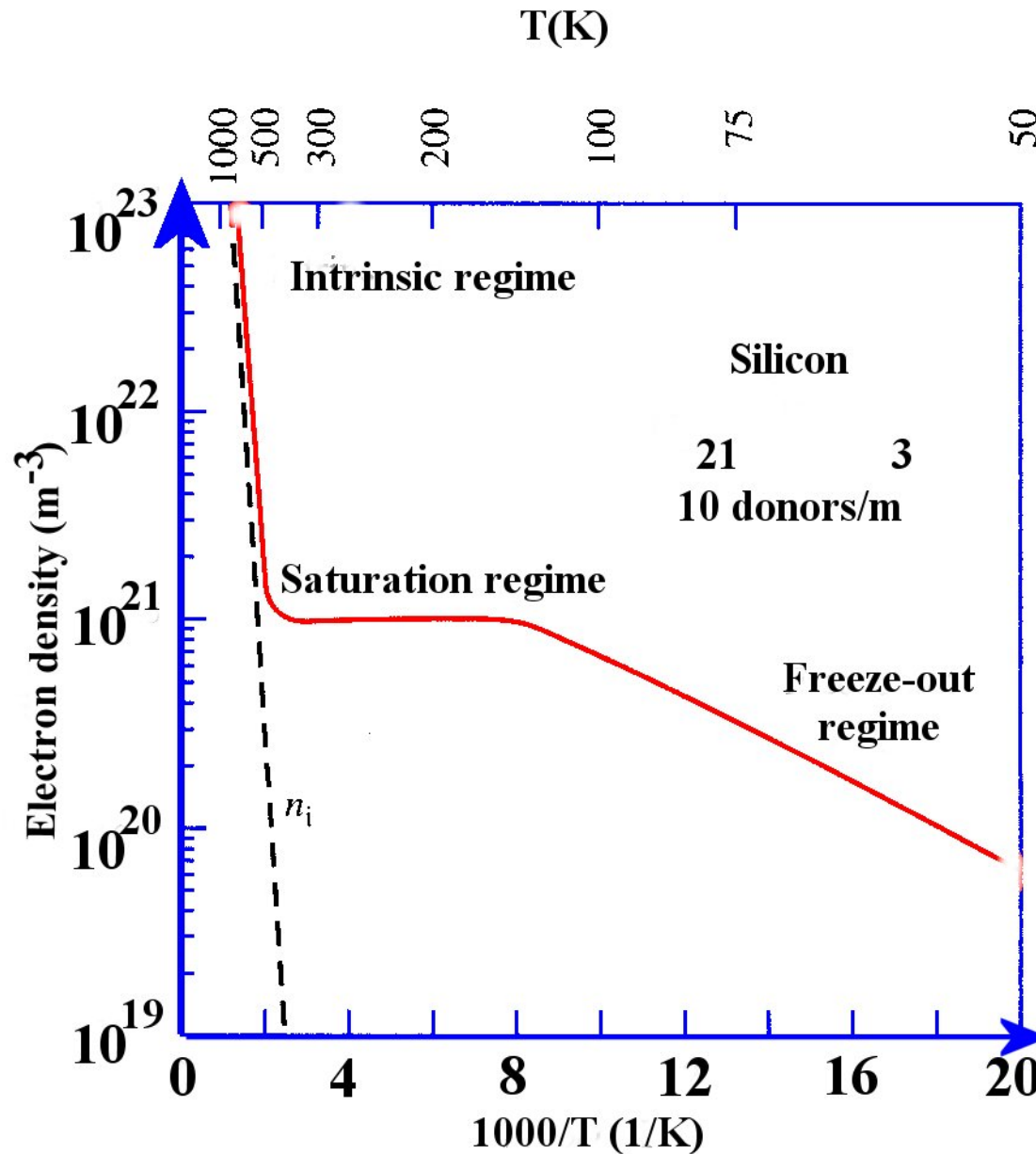
9.7 Carrier concentrations

9.7.1 Overview

Consider the electron density in an n-type semiconductor:



- At very low T , $n_e \propto e^{-(E_c - E_d)/(k_B T)}$ (**pinned μ**);
- At low T ($k_B T$ comparable with impurity binding energy) $n_e \propto e^{-(E_c - E_d)/(2k_B T)}$
- At intermediate T we exhaust all the impurities, but have not enough thermal energy to excite from the valence band – *saturation*
- At higher T we have $n_e \propto e^{-E_g/(2k_B T)}$.



Note that $n_e(T)n_h(T) = e^{-E_g/(k_B T)}n_c(T)n_v(T)$ irrespective of doping.

At room temperatures, $n_e n_h \approx 10^{38} \text{ m}^{-6}$ for Ge and 10^{33} m^{-6} for Si. So if there is no doping, $n_e = n_h \approx 3 \times 10^{16} \text{ m}^{-3}$ for Si. So to observe intrinsic behaviour at room temperature, need fewer carriers than this from impurities, a *concentration* of less than one part in 10^{12} of 10^{13} – unachievable.

9.7.2 Detailed results

At low temperatures, in an n-type material, if there are n_D donors per volume, we know that the number of ionized donors will be

$$n_D^+ = n_D \left[1 - \frac{1}{\exp((E_D - \mu)/(k_B T)) + 1} \right],$$

i.e. we compute the probability that the donor states will be empty. If we can assume that both

$$\mu - E_D \gg k_B T$$

and

$$E_c - \mu \gg k_B T$$

we can again use the Boltzmann expressions. But these require that μ lies between the donor levels and the conduction band, and these are only a few tens of meV apart, so this is only applicable at low T .

Then

$$n_e = \sqrt{n_D n_c(T)} e^{-(E_c - E_D)/(2k_B T)}.$$

9.8 Mobility and conductivity

If both electrons and holes are present, both contribute to the electrical conductivity:

$$\sigma = n_e e \mu_e + n_h e \mu_h.$$

In a doped material, one carrier type will be present in larger number at room temperatures – the *majority carrier*. The other is the *minority carrier*. At high T , the material behaves *intrinsically*, with roughly equal concentrations of electrons and holes. The main factors affecting the mobilities are scattering by charged impurities and phonon scattering. The real temperature dependences are complicated, but one can make rough estimates.

9.8.1 Scattering by charged impurities

Assume that a carrier is scattered when its potential energy in the field of the scatterer is similar to its kinetic energy. The potential energy, Coulombic, at a distance r

$$V \propto \frac{1}{r}.$$

The kinetic energy is thermal energy,

$$E \propto T$$

so we can define an effective radius of the scatterer as

$$r_s \propto \frac{1}{T}.$$

Hence we get a scattering cross-section, and a scattering probability,

$$p_{\text{scatt}} \propto \pi r_s^2 \propto T^{-2}.$$

The rate at which the carrier encounters scatterers is proportional to the carrier velocity

$$v \propto \sqrt{T},$$

so overall

$$p_{\text{scatt}} \propto T^{-3/2}.$$

9.8.2 Scattering by phonons

As in metals, the probability of interacting with a phonon is proportional to the number of phonons, which is proportional to T at room temperature. But the rate at which the carriers pass through the crystal is determined by their thermal velocity,

$$v \propto \sqrt{T},$$

so

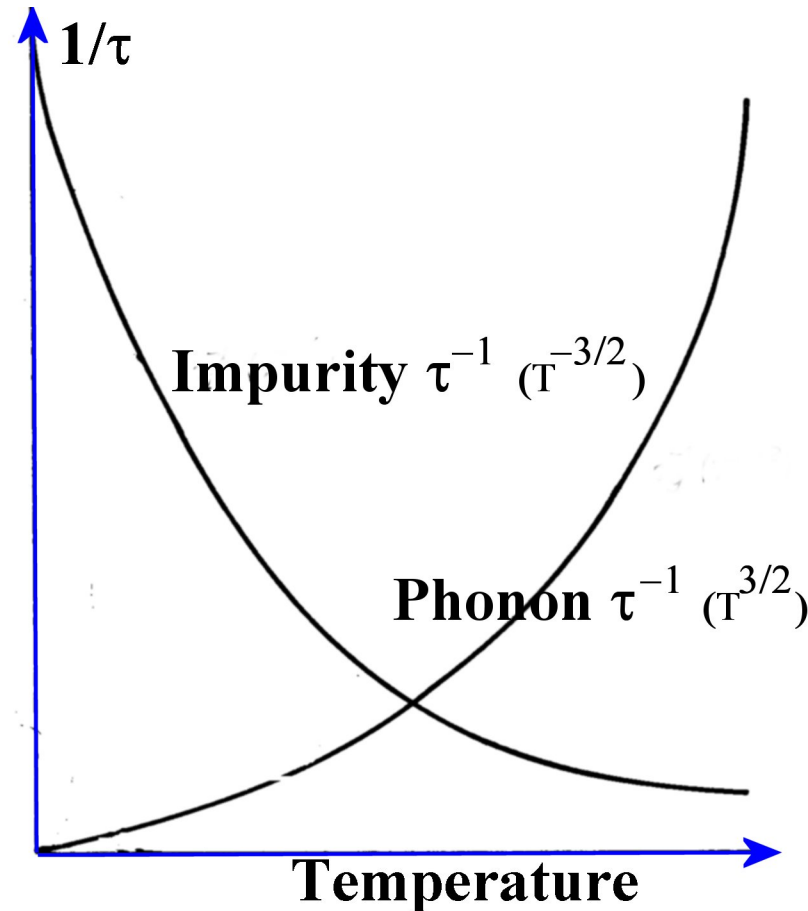
$$p_{\text{scatt}} \propto T^{3/2}.$$

Note the difference from metals – there the velocity of the carriers being scattered was the Fermi velocity, essentially independent of T .

9.8.3 Overall effect

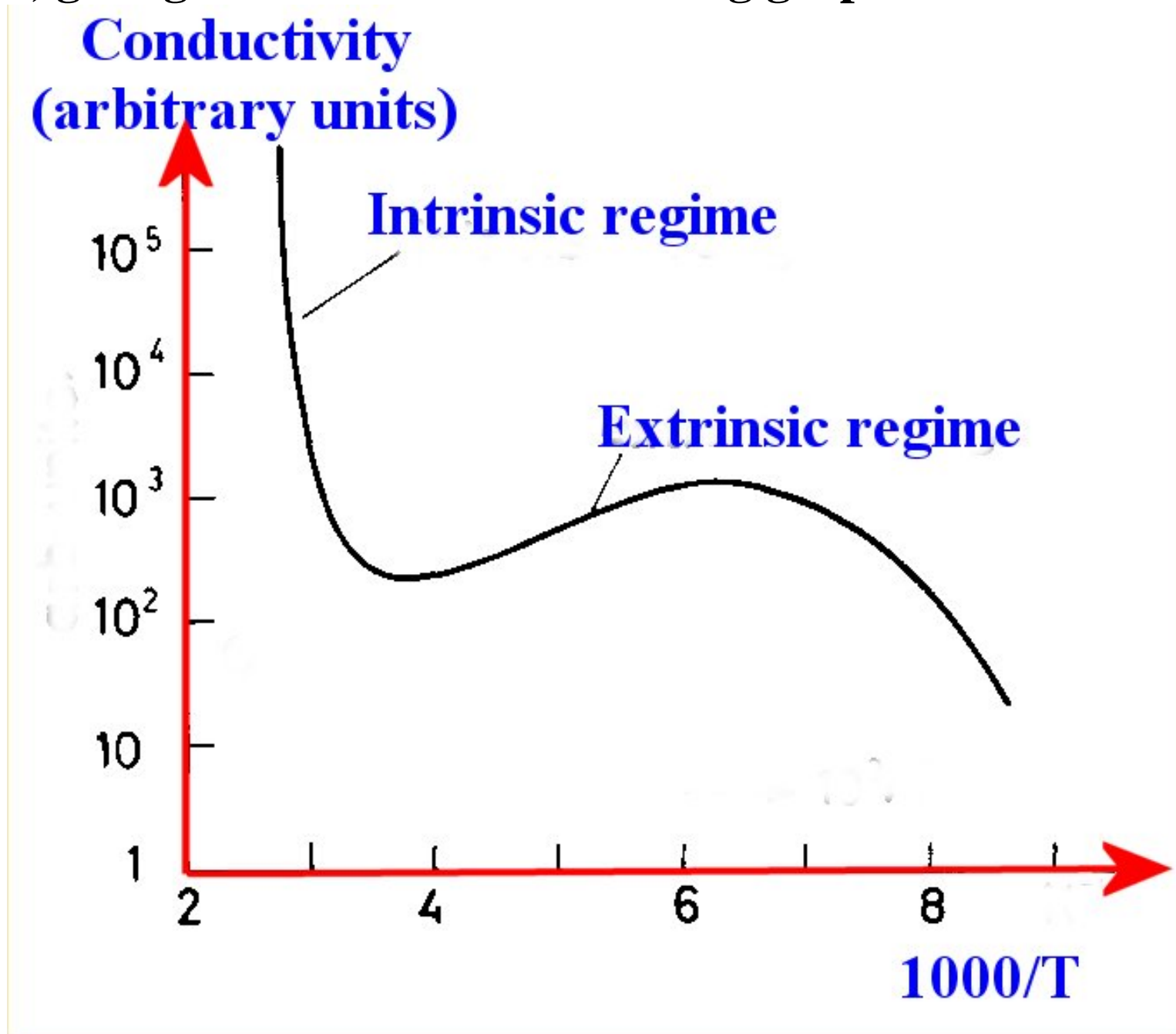
The graph shows the variation of the two contributions to $1/\tau$, and as usual

$$\frac{1}{\tau} = \frac{1}{\tau_{\text{def}}} + \frac{1}{\tau_{\text{phon}}}.$$



So the mobility peaks at intermediate temperatures – typically 100 to 200 K.

Then, to find the conductivity, we need to factor in the number of carriers, giving the result in the following graph.



9.8.4 Hall effect in semiconductors

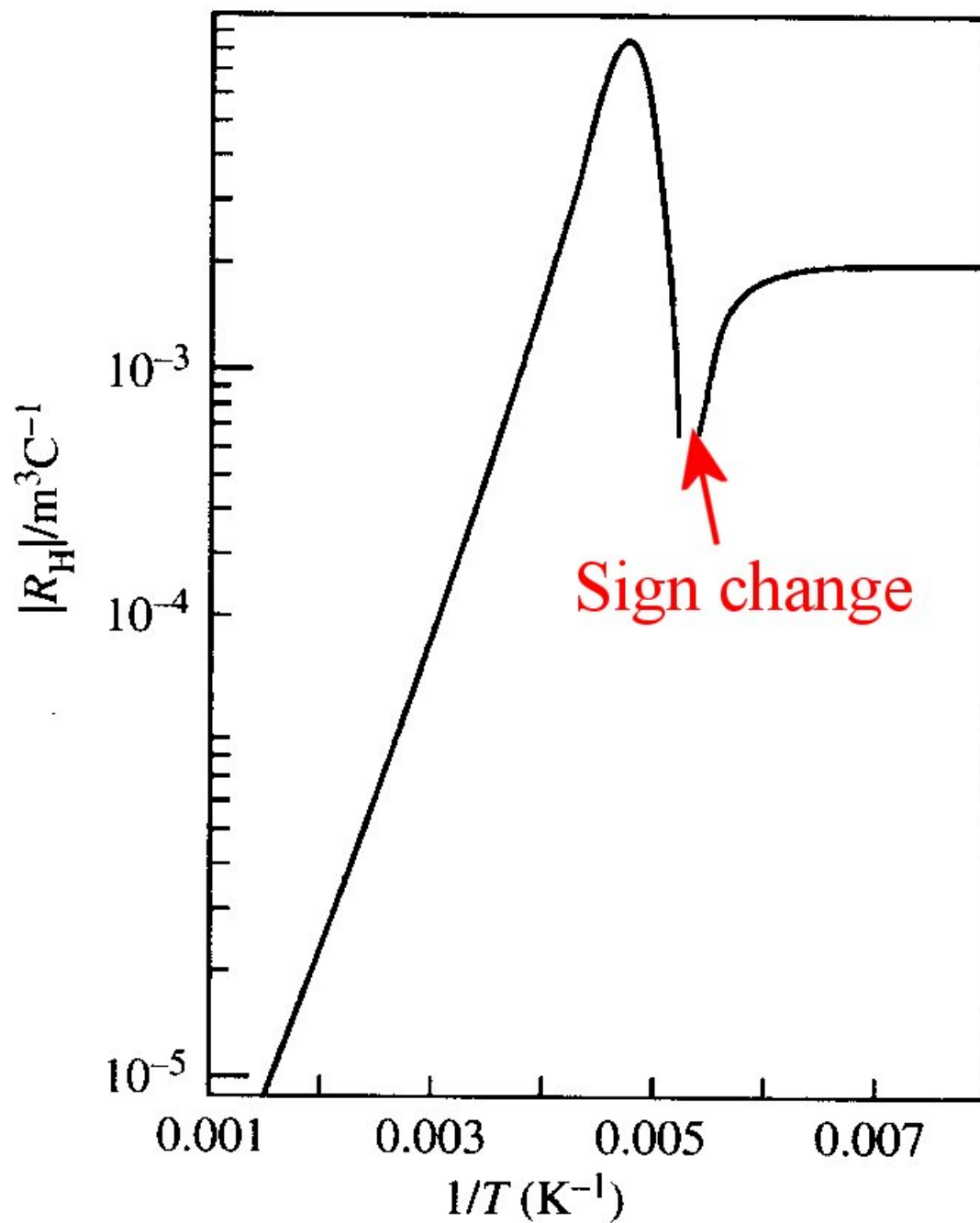
With more than one carrier type, the Hall coefficient is

$$R_H = \frac{1}{|e|} \frac{n_h \mu_h^2 - n_e \mu_e^2}{(n_h \mu_h + n_e \mu_e)^2}.$$

For a doped semiconductor, it is possible for the sign of the Hall coefficient to vary with T : for example, consider a p-type material with $\mu_e > \mu_h$

- **at low T , $R_H > 0$**
- **at high T , intrinsic behaviour gives $n_e = n_h$, but $\mu_e > \mu_h$ so $R_H < 0$**
- **temperature dependence of carrier concentration gives exponential dependence of R_H at high T**

Example: Hall coefficient in InSb.



9.8.5 Cyclotron resonance

In stronger magnetic fields \mathcal{B} , the carriers move in spirals about the field lines. For holes for example,

$$\frac{m_{\text{h}}^* v^2}{r} = \mathcal{B} e v,$$

so that the angular frequency $\omega_{\text{c}} = v/r$ is

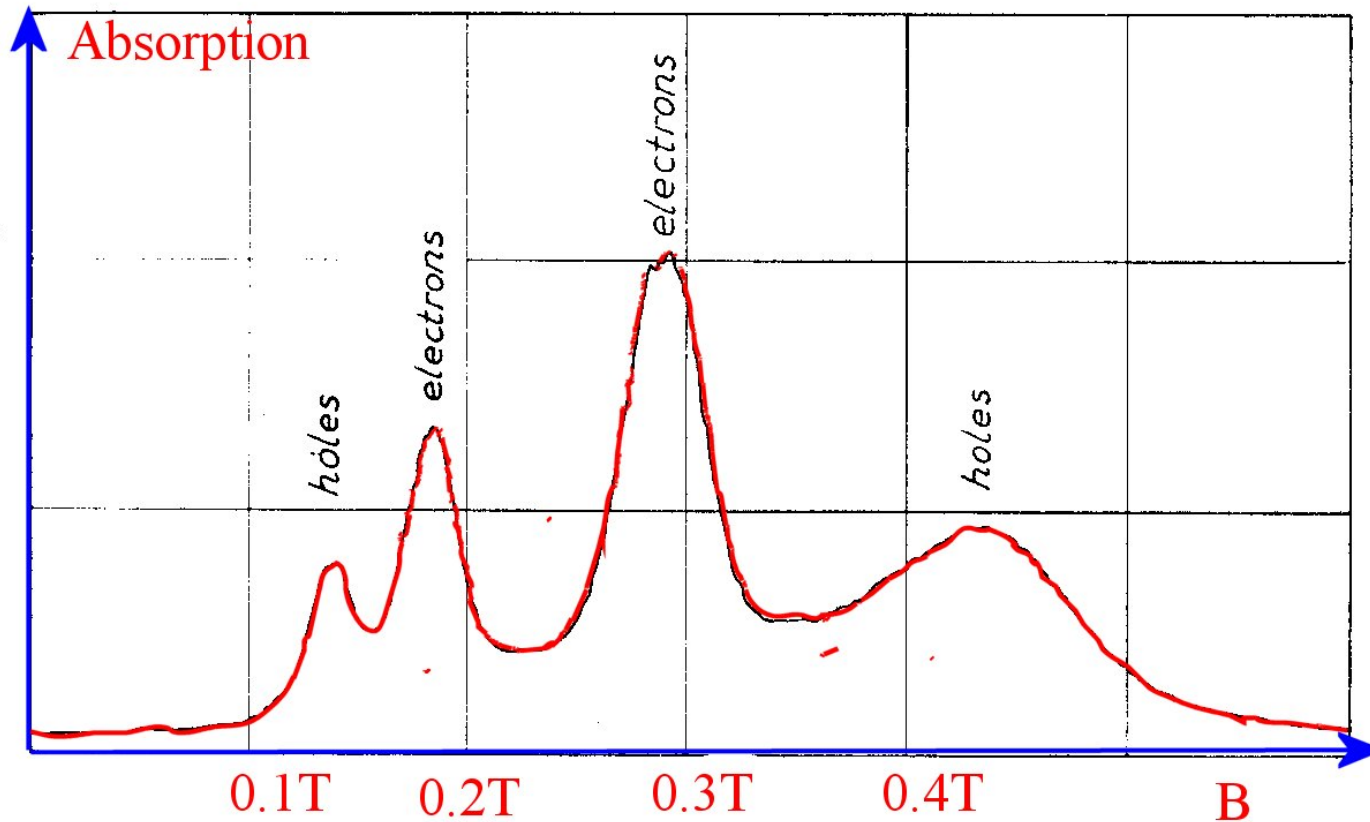
$$\omega_{\text{c}} = \frac{e\mathcal{B}}{m_{\text{h}}^*}.$$

This is the *cyclotron frequency*: electromagnetic radiation of that frequency can be absorbed, giving a measurement of m_{h}^* . We do not expect to be able to detect this cyclotron resonance unless the carrier completes most of an orbit before being scattered,

$$\omega_{\text{c}}\tau \sim 1.$$

This dictates the range of frequency, and hence field, to use. Typically at room T use infrared, at liquid helium T use microwaves.

Cyclotron resonance in Si at 24 GHz at 4 K.



Note that we have heavy holes and light holes, but for electrons the constant energy surfaces are ellipsoids, so the effective mass is different for different directions. There is a vast array of beautiful experiments which explore details of Fermi surfaces, which we have no time to explore in this course.

9.9 Carrier diffusion and recombination

Suppose we have a p-type semiconductor, i.e.

$$n_h \gg n_e. \quad (1)$$

Create a local excess of minority carriers (electrons)

- with radiation, when $\Delta n_e = \Delta n_h$ automatically, or
- by using a contact, when electrical neutrality will ensure $\Delta n_e = \Delta n_h$.

But because of equation 1

$$\frac{\Delta n_e}{n_{e0}} \gg \frac{\Delta n_h}{n_{h0}},$$

so the change from equilibrium concentration (n_{e0} or n_{h0}) is much greater for the minority carriers.

9.9.1 Recombination

Electrons and holes annihilate, mainly at deep traps or surfaces. The recombination (annihilation) rate is proportional to the product of the concentrations:

$$R' = cn_en_h = c(n_{e0} + \Delta n_e)(n_{h0} + \Delta n_h).$$

But we know that in equilibrium we have dynamic equilibrium with thermal generation equal to recombination $cn_{e0}n_{h0}$, so the recombination caused by the excess carriers is

$$\begin{aligned} R &= c(n_{e0} + \Delta n_e)(n_{h0} + \Delta n_h) - cn_{e0}n_{h0} \\ &= cn_{e0}n_{h0} \left(\frac{\Delta n_e}{n_{e0}} + \frac{\Delta n_h}{n_{h0}} + \frac{\Delta n_h \Delta n_e}{n_{h0} n_{e0}} \right) \\ &= n_{e0}n_{h0} \frac{\Delta n_e}{n_{e0}}, \end{aligned}$$

keeping only the largest term. Thus

$$R = n_{h0}\Delta n_e,$$

the recombination rate is proportional to the concentration of excess minority carriers.

If we write

$$R = -\frac{d\Delta n_e}{dt} = \frac{\Delta n_e}{\tau_e},$$

then

$$\Delta n_e(t) = \Delta n_e(0)e^{-t/\tau_e}.$$

9.9.2 Diffusion

Suppose we inject excess minority carriers at some point: this will set up a carrier concentration gradient, and carriers will diffuse. As they carry charge, this will give an electric current density. For holes

$$J = -|e|D_h \nabla n_h,$$

the negative sign accounting for diffusion *down* the gradient. D_h is the diffusion constant. The rate of increase of hole density in a slice at x in one dimension is

$$\frac{\partial}{\partial x} \left(-D_h \frac{\partial n_h}{\partial x} \right) = -D_h \frac{\partial^2 n_h}{\partial x^2},$$

which in a steady state is balanced by recombination loss so

$$D_h \frac{\partial^2 n_h}{\partial x^2} = \frac{n_h - n_{h0}}{\tau_h}.$$

This is equivalent to

$$\frac{\partial^2 \Delta n_h}{\partial x^2} = \frac{\Delta n_h}{D_h \tau_h}$$

or

$$\Delta n_h(x) = \Delta n_h(0) e^{-x/l_h},$$

where

$$l_h = \sqrt{D_h \tau_h}$$

is called the *hole diffusion length*. Of course, for electrons there are exactly similar expressions. Note that the diffusion constant and the mobility are related by the *Einstein relations*

$$\mu_h = \frac{eD_h}{k_B T} \quad \mu_e = \frac{eD_e}{k_B T}.$$