

Solid State Physics

SEMICONDUCTORS - IV

Lecture 25

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9.9 Carrier diffusion and recombination

Suppose we have a p-type semiconductor, i.e.

$$n_h \gg n_e. \quad (1)$$

Create a local excess of minority carriers (electrons)

- with radiation, when $\Delta n_e = \Delta n_h$ automatically, or
- by using a contact, when electrical neutrality will ensure $\Delta n_e = \Delta n_h$.

But because of equation 1

$$\frac{\Delta n_e}{n_{e0}} \gg \frac{\Delta n_h}{n_{h0}},$$

so the change from equilibrium concentration (n_{e0} or n_{h0}) is much greater for the minority carriers.

9.9.1 Recombination

Electrons and holes annihilate, mainly at deep traps or surfaces. The recombination (annihilation) rate is proportional to the product of the concentrations:

$$R' = cn_en_h = c(n_{e0} + \Delta n_e)(n_{h0} + \Delta n_h).$$

But we know that in equilibrium we have dynamic equilibrium with thermal generation equal to recombination $cn_{e0}n_{h0}$, so the recombination caused by the excess carriers is

$$\begin{aligned} R &= c(n_{e0} + \Delta n_e)(n_{h0} + \Delta n_h) - cn_{e0}n_{h0} \\ &= cn_{e0}n_{h0} \left(\frac{\Delta n_e}{n_{e0}} + \frac{\Delta n_h}{n_{h0}} + \frac{\Delta n_h \Delta n_e}{n_{h0} n_{e0}} \right) \\ &= cn_{e0}n_{h0} \frac{\Delta n_e}{n_{e0}}, \end{aligned}$$

keeping only the largest term. Thus

$$R = cn_{h0}\Delta n_e,$$

the recombination rate is proportional to the concentration of excess minority carriers.

If we write cn_{h0} as $1/\tau_e$,

$$R = -\frac{d\Delta n_e}{dt} = \frac{\Delta n_e}{\tau_e},$$

then

$$\Delta n_e(t) = \Delta n_e(0)e^{-t/\tau_e},$$

where τ_e is the electron (minority carrier) recombination time.

9.9.2 Diffusion

Suppose we inject excess minority carriers at some point: this will set up a carrier concentration gradient, and carriers will diffuse. As they carry charge, this will give an electric current density. For electrons

$$J_{e,\text{diff}} = -(-|e|)D_e \nabla n_e,$$

one negative sign accounting for diffusion *down* the gradient, the other being the sign of the carrier charge. D_e is the diffusion constant. The rate of increase of electron density in a slice at x in one dimension is

$$\frac{\partial}{\partial x} \left(-D_e \frac{\partial n_e}{\partial x} \right) = -D_e \frac{\partial^2 n_e}{\partial x^2},$$

which in a steady state is balanced by recombination loss so

$$D_e \frac{\partial^2 n_e}{\partial x^2} = \frac{n_e - n_{e0}}{\tau_e}.$$

This is equivalent to

$$\frac{\partial^2 \Delta n_e}{\partial x^2} = \frac{\Delta n_e}{D_e \tau_e}$$

or

$$\Delta n_e(x) = \Delta n_e(0) e^{-x/l_e}, \quad (2)$$

where

$$l_e = \sqrt{D_e \tau_e}$$

is called the *electron diffusion length*. Of course, for holes there are exactly similar expressions. Note that the diffusion constant and the mobility are related by the *Einstein relations*

$$\mu_h = \frac{eD_h}{k_B T} \quad \mu_e = \frac{eD_e}{k_B T}.$$

9.9.3 Electric current

In general, there can be four contributions to electric current in a semiconductor:

- **electron drift:** $J_{e,\text{drift}} = n_e \mu_e |e| \mathcal{E}$
- **hole drift:** $J_{h,\text{drift}} = n_h \mu_h |e| \mathcal{E}$
- **electron diffusion:** $J_{e,\text{diff}} = |e| D_e \nabla n_e$
- **hole diffusion:** $J_{h,\text{diff}} = -|e| D_h \nabla n_h$

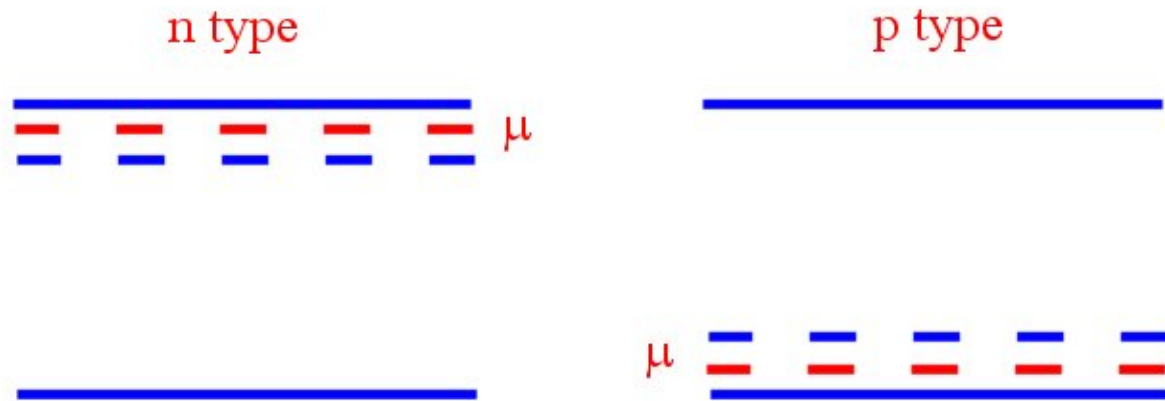
Carrier mobilities, $\text{m}^2\text{V}^{-1}\text{s}^{-1}$ at room T

	Electrons	Holes		Electrons	Holes
Diamond	0.018	0.012	GaAs	0.080	0.003
Si	0.014	0.005	GaSb	0.050	0.010
Ge	0.036	0.018	PbS	0.006	0.006
InSb	0.300	0.005	PbSe	0.010	0.009
InP	0.045	0.001	AlSb	0.009	0.004

9.10 Heterojunctions

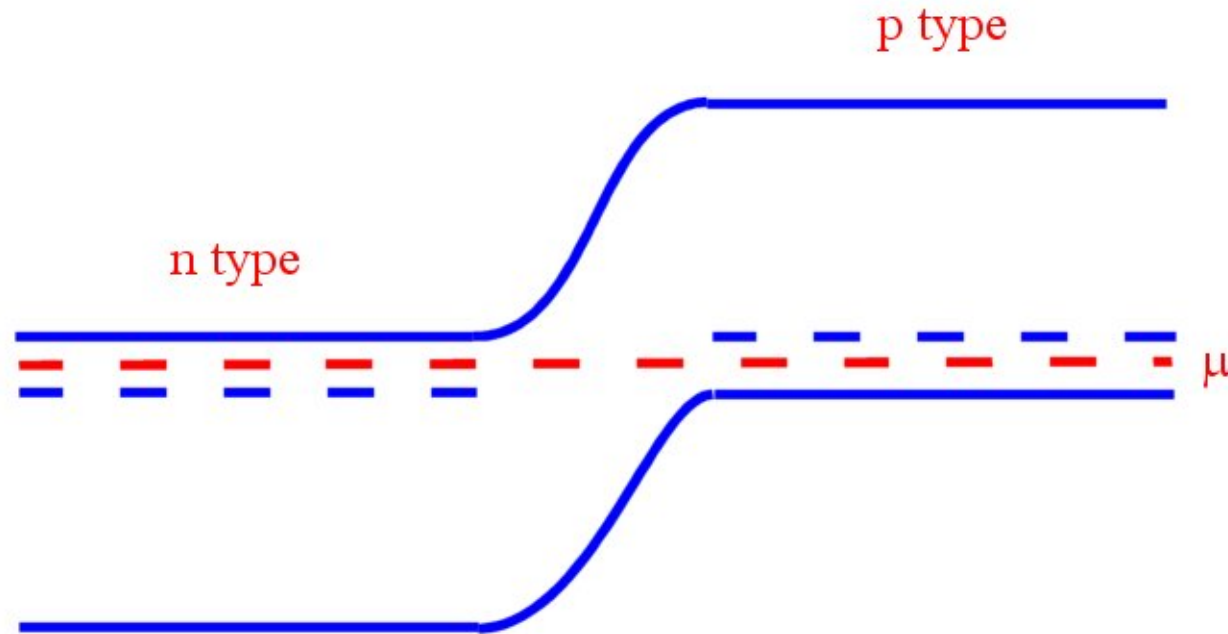
Most important semiconductor devices depend on having differently-doped materials in contact. In practice, these are made by ion implantation or diffusion, giving relatively smooth dopant concentration variations – but we assume sharp boundaries.

Consider an n-type and a p-type material.

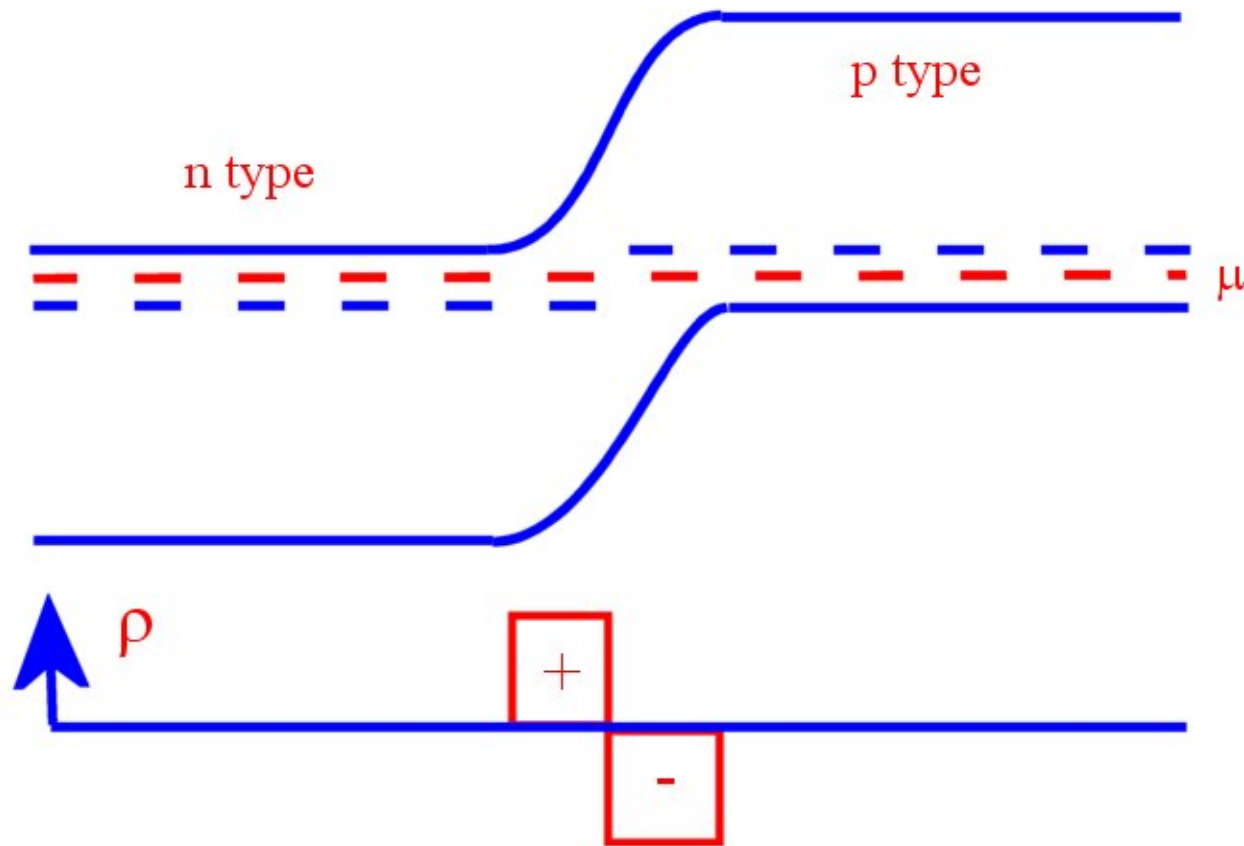


When they are separated, their chemical potentials are roughly E_g apart.

When they are in contact and in equilibrium the chemical potential must be constant throughout.



This can happen if the p-type region becomes negative, raising the potential for electrons, and the n-type becomes positive.



We assume this happens by ionising the impurities: the electrons released from donors in a region near the interface go to acceptors near the interface.

Suppose a region of thickness x_D with donor density n_D and a region of thickness x_A with acceptor density n_A are ionised. The ionisation is assumed to be total within this *depletion zone*, where there are practically no free carriers. In a region with charge density ρ Poisson's equation tells us the electric field is given by

$$\frac{d\mathcal{E}}{dx} = \frac{\rho}{\epsilon_0\epsilon_r}.$$

If the field is zero outside the depletion zone, $\mathcal{E} = 0$ at $x = -x_D$, and in the depletion zone in the n-type material $\rho = |e|n_D$, so

$$\mathcal{E} = \frac{n_D|e|}{\epsilon_0\epsilon_r}(x + x_D).$$

As the potential \mathcal{V} is related to the field by $\mathcal{E} = -d\mathcal{V}/dx$ we have

$$\mathcal{V} = V(-x_D) - \frac{n_D|e|}{2\epsilon_0\epsilon_r}(x + x_D)^2.$$

Similarly in the p-type depletion zone

$$\mathcal{E} = \frac{n_A|e|}{\epsilon_0\epsilon_r}(x_A - x),$$

and we have a total change in potential across the depletion region

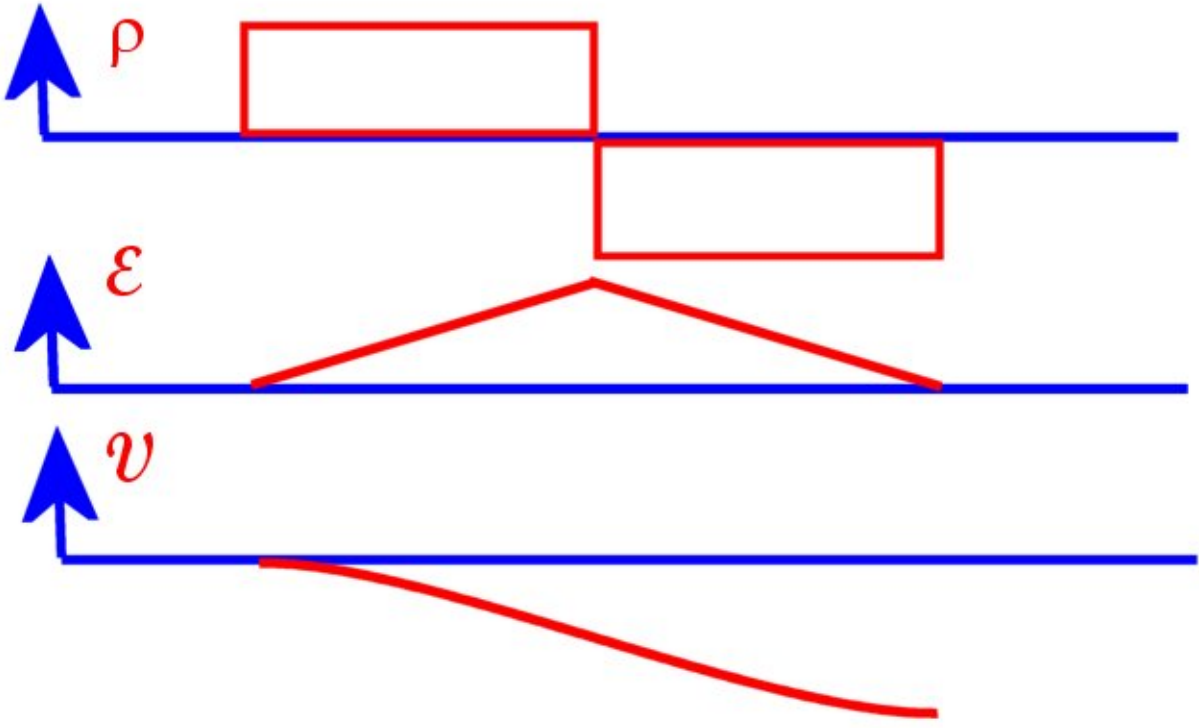
$$\Delta\mathcal{V} = \frac{|e|}{2\epsilon_0\epsilon_r} \left(n_D x_D^2 + n_A x_A^2 \right).$$

This must give a voltage equal to the band gap. Putting $n_D x_D = n_A x_A$ (which ensures continuity of \mathcal{V} at the interface) we find

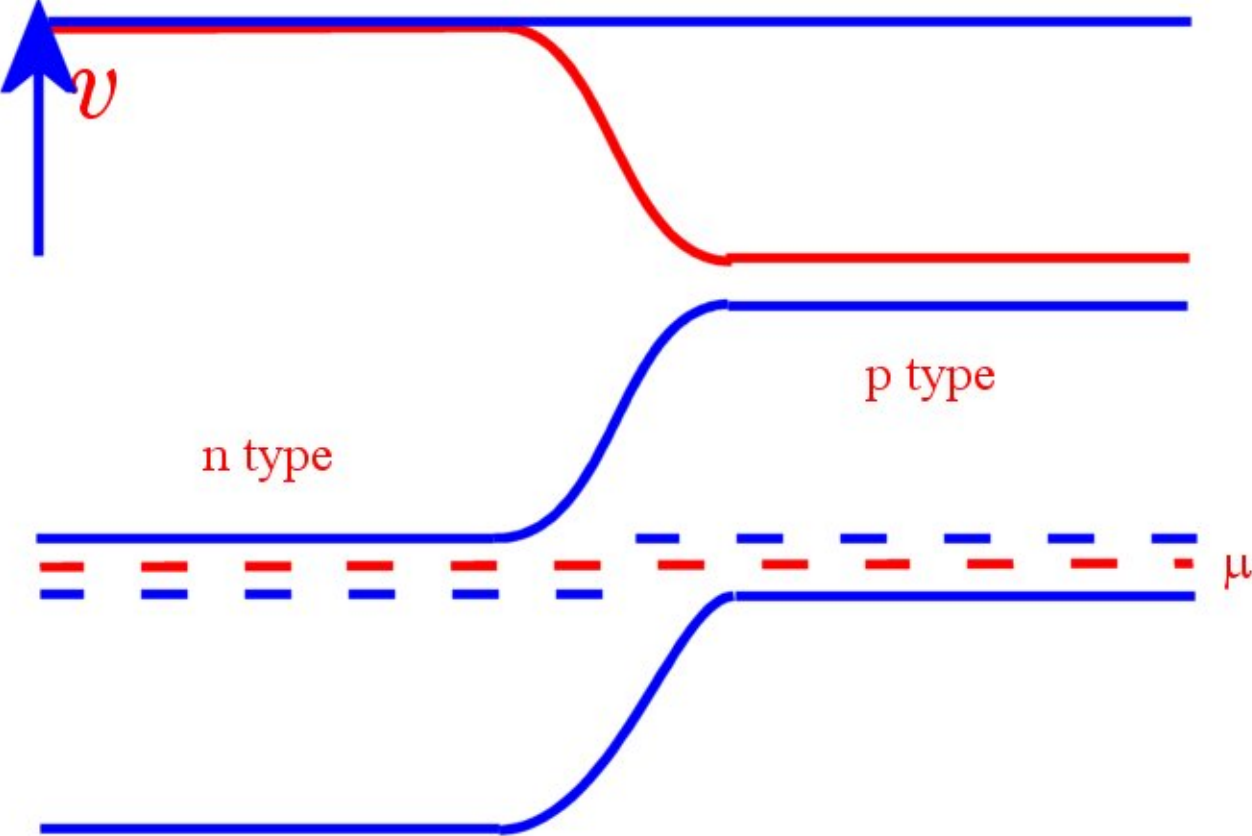
$$x_D = \sqrt{\frac{2\epsilon_0\epsilon_r}{n_D|e|} V \left(\frac{n_A}{n_A + n_D} \right)},$$

giving, with a band gap of 0.5 V and dopant concentrations of about 10^{23} m^{-3} , depletion layer widths of about $1 \mu\text{m}$.

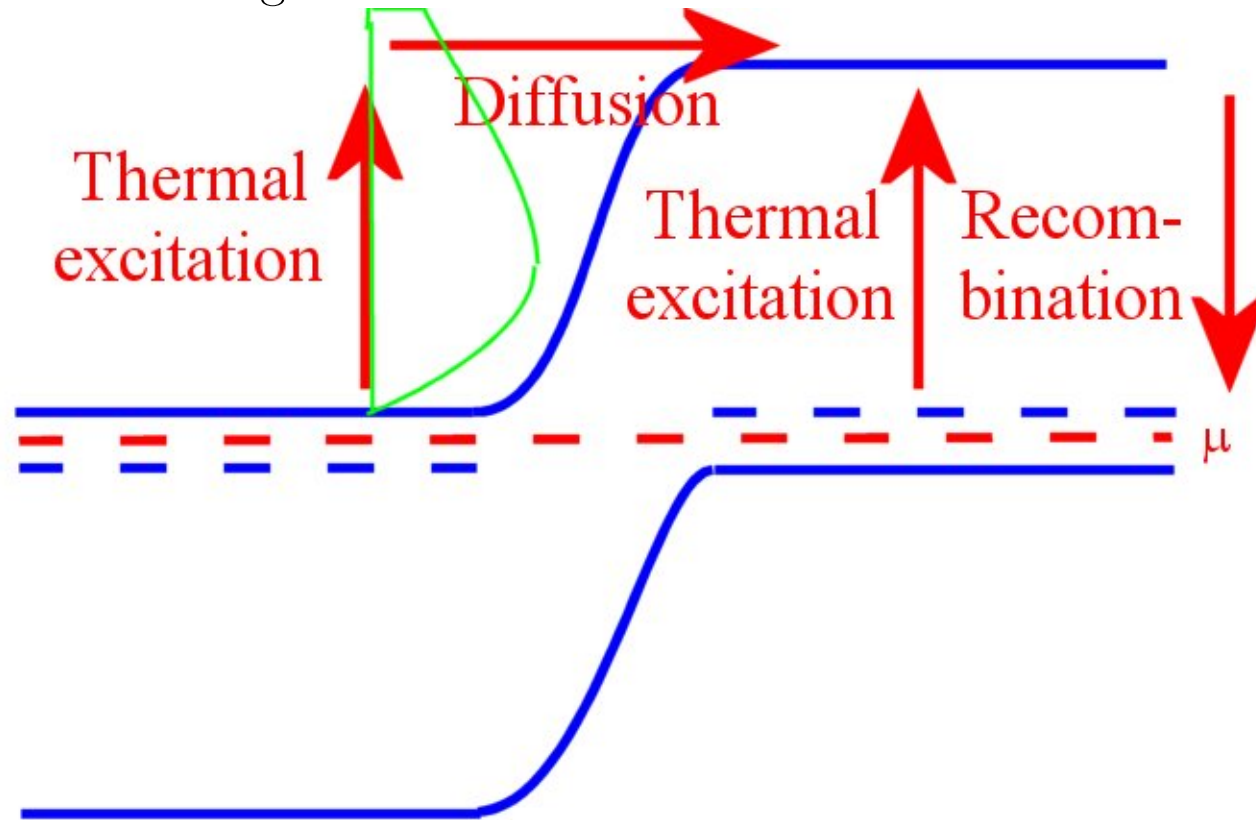
The charge densities, fields, and potential are shown below.



Here is the relationship between the potential through the junction and the band structure.

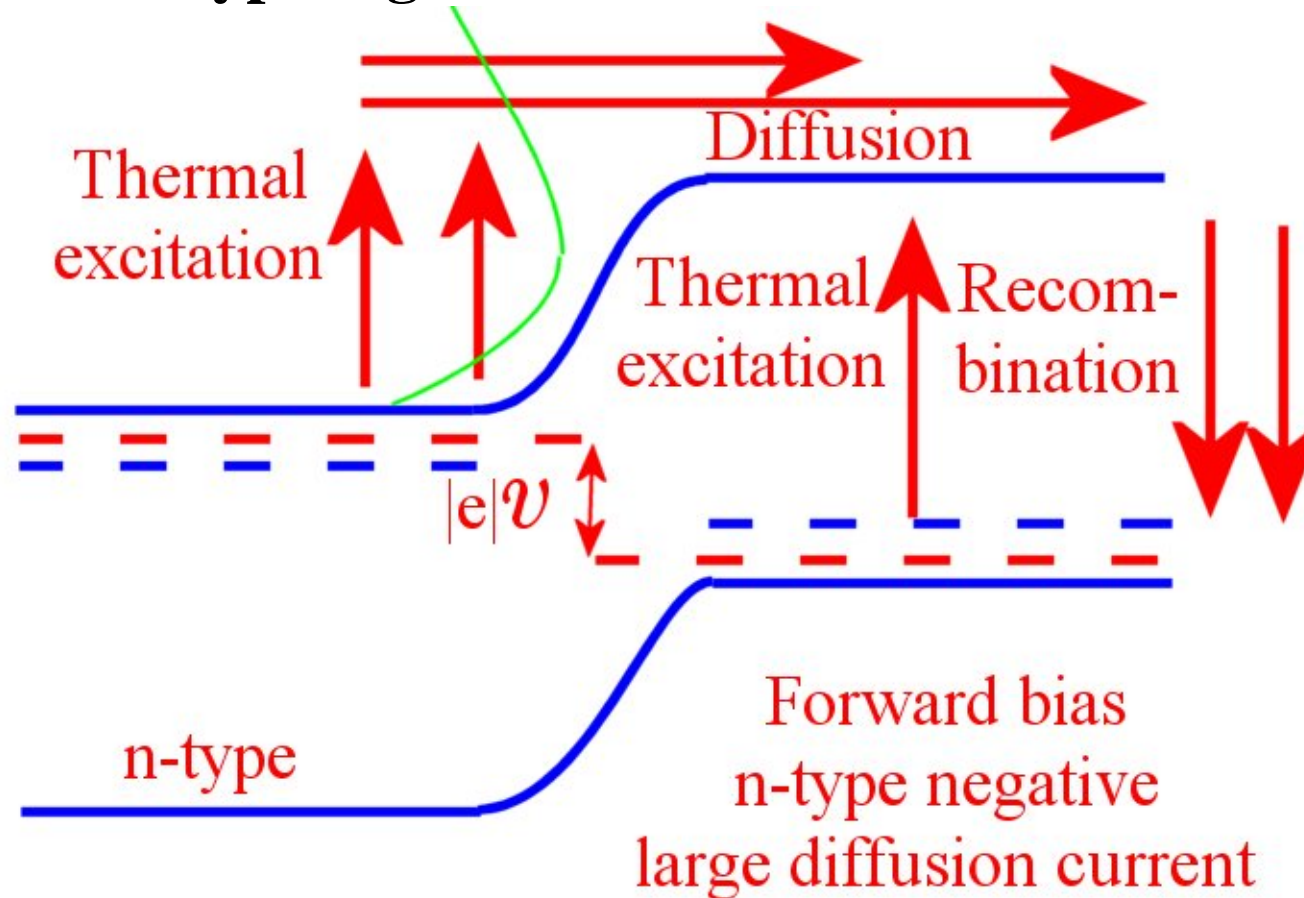


In equilibrium, we can assume there are practically no electrons on the p-type side. On the n-type side the fraction of the electrons with enough energy to move to the p-type side will vary as $\exp(-E_g/k_B T)$ (those with energy E_g above the bottom of the conduction band.)



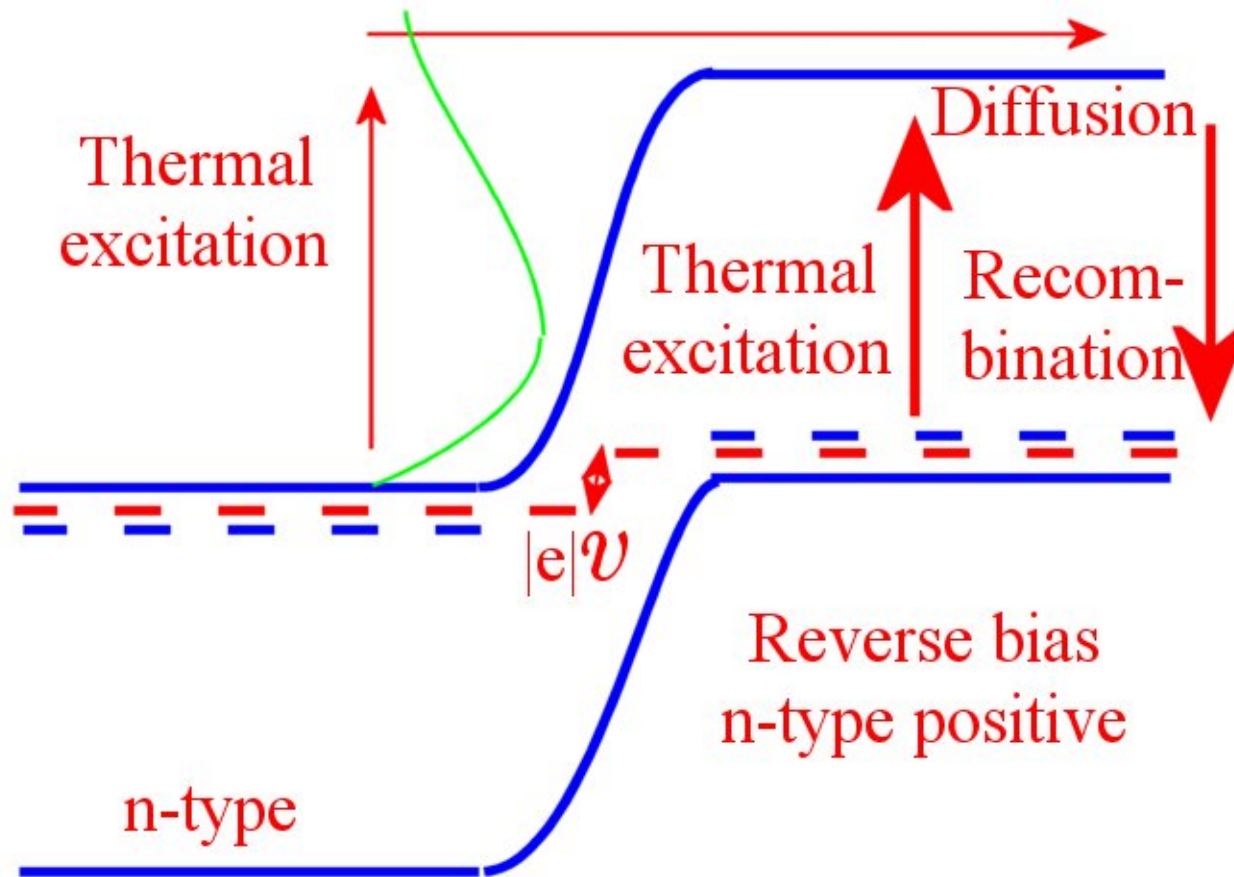
Once they are in the p-type material, these diffuse a distance l_e before they recombine.

in equilibrium, these are balanced by a flow of thermally-generated electrons in the p-type region, which roll down the potential energy surface into the n-type region.



But forward bias the junction, raising the energy of the electrons in the n-type region by eV , and the number passing from n to p is increased by a factor $\exp(eV/k_B T)$.

Under reverse bias, the number of electrons flowing from n to p is reduced, and as there are hardly any electrons in the p-type the reverse current is very low.



Thus the number of electrons very close to the junction on the p-type side will be

$$n_{e0p} + An_{e0p} \left[e^{e\mathcal{V}/k_B T} - 1 \right],$$

where A is a diffusion parameter from p to n. The extra concentration of electrons on the p side varies as (equation 2

$$\Delta n_e(x) = \Delta n_e(0)e^{-x/l_e}, \quad (3)$$

and the current is given by the product of the diffusion constant and the concentration gradient, so for forward bias

$$J = \frac{eD_eAn_{e0p}}{l_e} \left[e^{e\mathcal{V}/k_B T} - 1 \right],$$

and for reverse bias

$$J = \frac{eD_eAn_{e0p}}{l_e} \left[1 - e^{e\mathcal{V}/k_B T} \right].$$

Of course, the electrons crossing the barrier will be supplied by a drift current in the n-type material. There will be a hole diffusion current in the n-type material too.

The p-n junction is a *rectifier*.

