MAGNETISM I

Lecture 27

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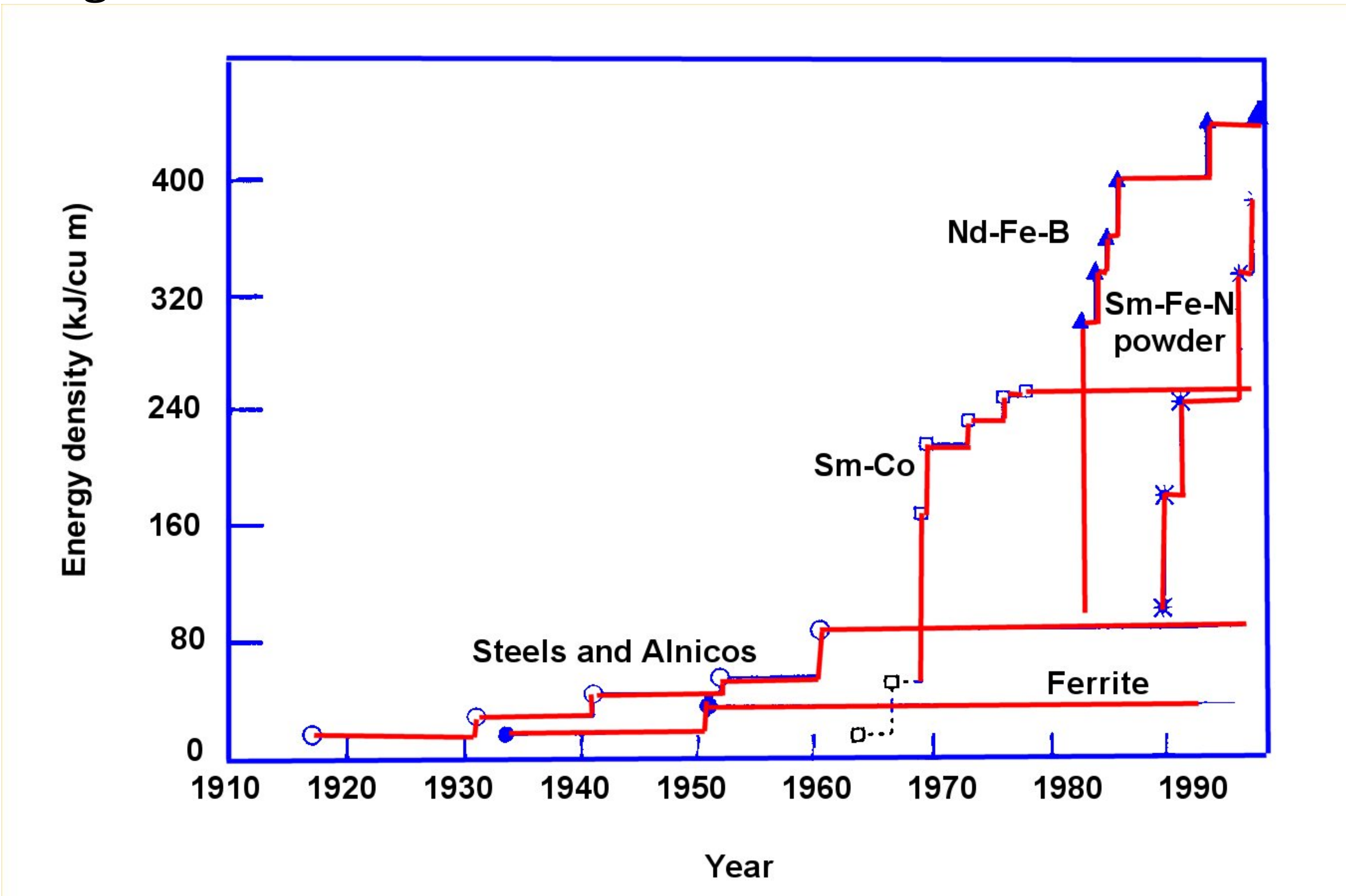
10 Magnetic Materials

Magnet technology has made enormous advances in recent years – without the reductions in size that have come with these advances many modern devices would be impracticable.

Same magnetic energy - fraction of a Joule.

The important quantity for many purposes is the energy density of the magnet.



10.1 Magnetic properties - reminder

There are two fields to consider, the magnetic field $\mathcal{H}$ which is generated by currents according to Ampère's law, and the magnetic induction, or magnetic flux density, $\mathcal{B}$, which gives the torque experienced by a dipole moment m as $\mathrm{G} = \mathrm{m} \times \mathcal{B}$. $\mathcal{H}$ is measured in $\mathrm{A\ m^{-1}}$ (Oersteds in old units), $\mathcal{B}$ in $\mathrm{Wb\ m^{-2}}$ or T (Gauss in old units). In free space, $\mathcal{B} = \mu_0 \mathcal{H}$. In a material

$$\begin{aligned} \mathcal{B} &= \mu_0(\mathcal{H} + \mathcal{M}) \\ &= \mu_0 \mu_{\mathrm{r}} \mathcal{H} \\ &= \mu_0(1 + \chi)\mathcal{H}, \end{aligned}$$

where μ_{r} is the relative permeability, χ is the magnetic susceptibility, which is a dimensionless quantity.

Note, though, that χ is sometimes tabulated as the *molar susceptibility*

$$\chi_{\mathrm{m}} = V_{\mathrm{m}}\chi,$$

where V_{m} is the volume occupied by one mole, or as the *mass susceptibility*

$$\chi_{\mathrm{g}} = \frac{\chi}{\rho},$$

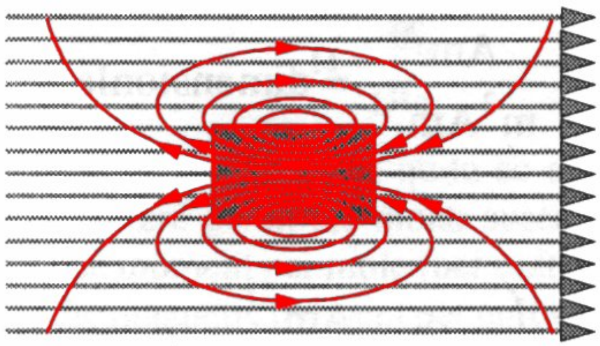
where ρ is the density. $\mathcal{M}$, the magnetisation, is the dipole moment per unit volume.

$$\mathcal{M} = \chi\mathcal{H}.$$

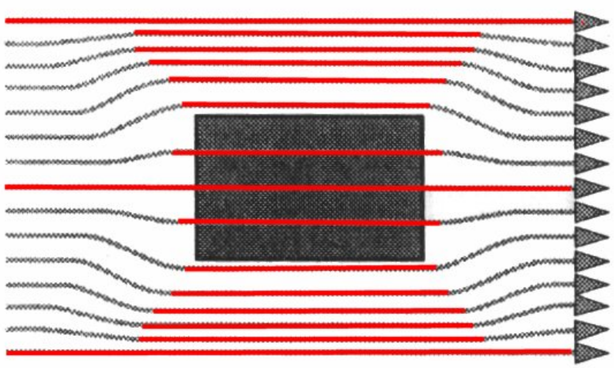
In general, μ_{r} (and hence χ) will depend on position and will be tensors (so that $\mathcal{B}$ is not necessarily parallel to $\mathcal{H}$). Even worse, some materials are non-linear, so that μ_{r} and χ are field-dependent.

A diamagnetic sample in a magnetic field

The lines of flux of the field due to the "bar magnet" tend to *decrease* the field inside the material and *increase* the field on either side of the material

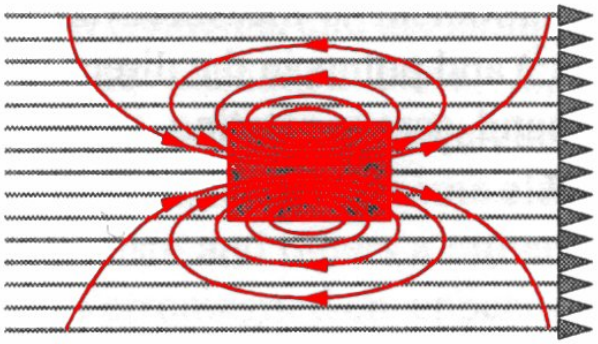


The flux density inside the material is *less* than the flux density of the applied field, i.e. the sample has "repelled" flux lines

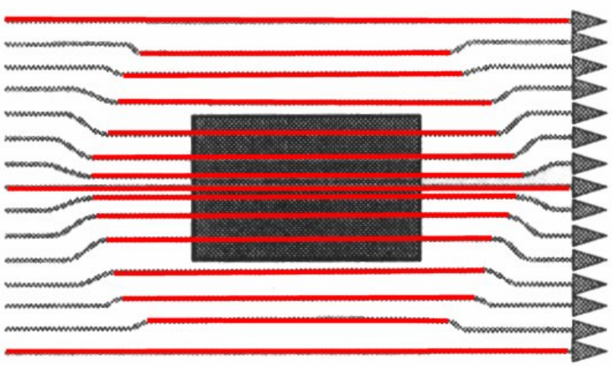


A paramagnetic sample in a magnetic field

The lines of flux of the field due to the "bar magnet" tend to *increase* the field inside the material and *decrease* the field on either side of the material



The flux density inside the material is *greater* than the flux density of the applied field, i.e. the sample has "concentrated" flux lines.



The effects are *highly exaggerated* in these diagrams.

10.2 Measuring magnetic properties

10.2.1 Force method

Uses energy of induced dipole

$$E = -\frac{1}{2}m\mathcal{B} = -\frac{1}{2}\mu_0\chi V\mathcal{H}^2,$$

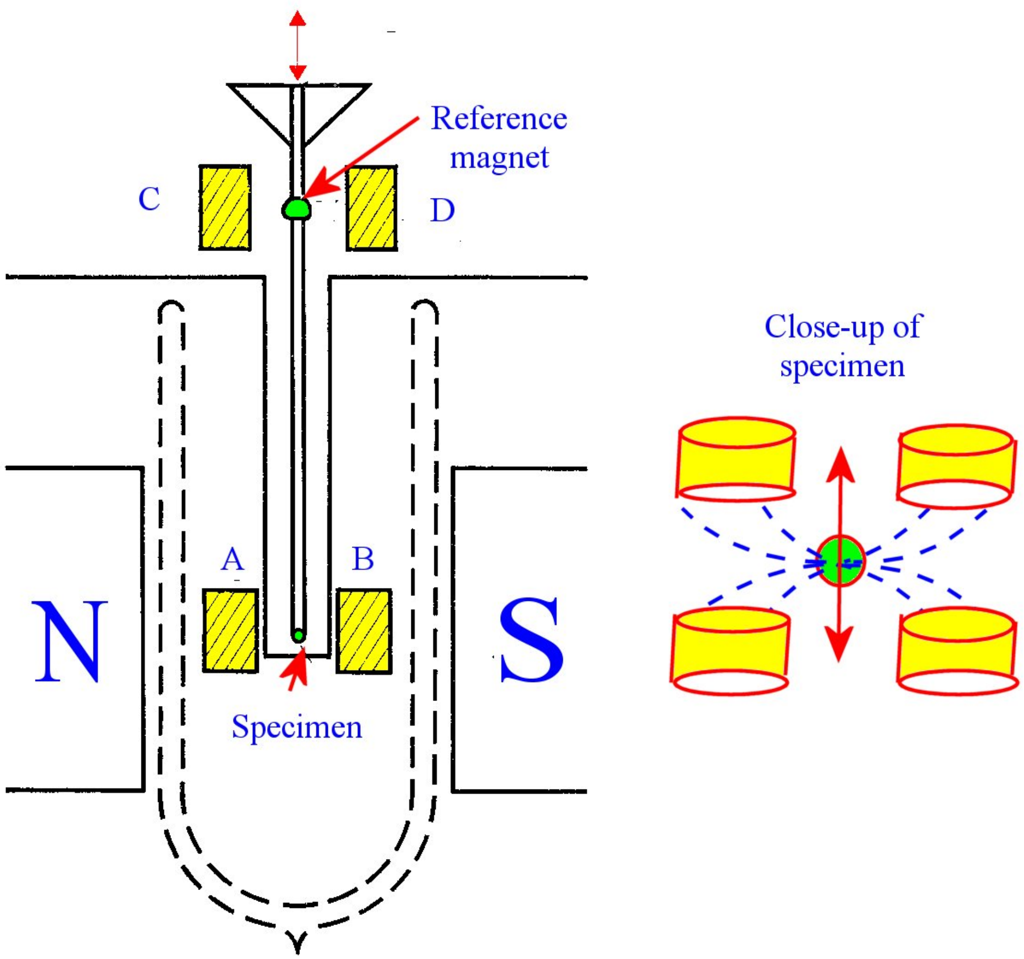
so in an inhomogeneous field

$$F = -\frac{\mathrm{d}E}{\mathrm{d}x} = \frac{1}{2}\mu_0 V\chi\frac{\mathrm{d}\mathcal{H}^2}{\mathrm{d}x} = \mu_0 V\chi\mathcal{H}\frac{\mathrm{d}\mathcal{H}}{\mathrm{d}x}.$$

Practically:

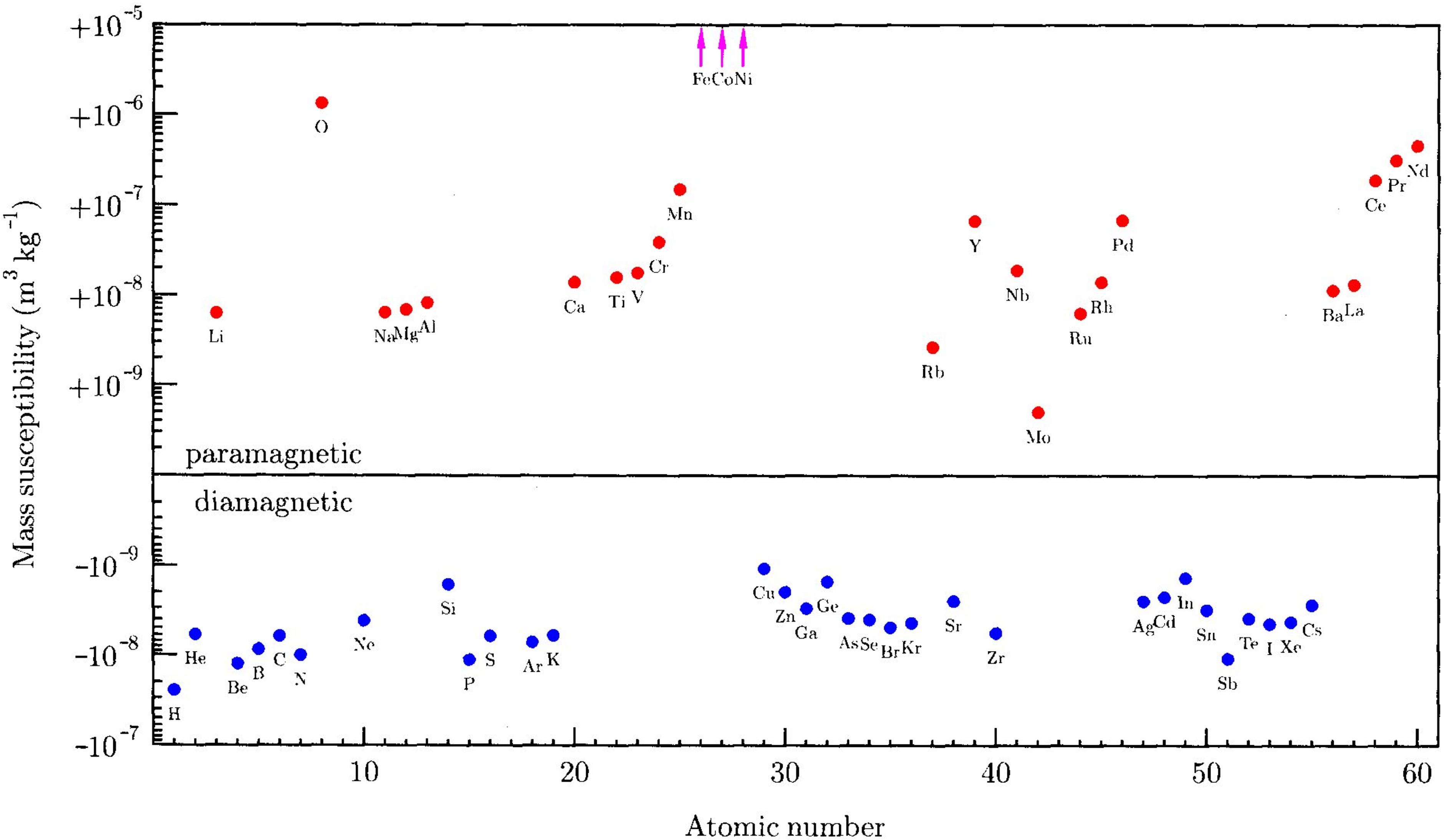
- **set up large uniform $\mathcal{H}$;**
- **superpose linear gradient with additional coils**
- **vary second field sinusoidally and use lock-in amplifier to measure varying force**

10.2.2 Vibrating Sample magnetometer



- oscillate sample up and down
- measure emf induced in coils A and B
- compare with emf in C and D from known magnetic moment
- hence measured sample magnetic moment

10.3 Experimental data



In the first 60 elements in the periodic table, the majority have negative susceptibility – they are *diamagnetic*.

10.4 Diamagnetism

Classically, we have Lenz's law, which states that the action of a magnetic field on the orbital motion of an electron causes a back-emf which opposes the magnetic field which causes it. Frankly, this is an unsatisfactory explanation, but we cannot do better until we have studied the inclusion of magnetic fields into quantum mechanics using magnetic vector potentials. Imagine an electron in an atom as a charge e moving clockwise in the x-y plane in a circle of radius a, area A, with angular velocity ω. This is equivalent to a current

$$I = \text{charge}/\text{time} = e\omega/(2\pi),$$

so there is a magnetic moment

$$\mu = IA = e\omega a^2/2.$$

The electron is kept in this orbit by a central force

$$F = m_\mathrm{e}\omega^2 a.$$

Now if a flux density $\mathcal{B}$ is applied in the z direction there will be a Lorentz force giving an additional force along a radius

$$\Delta F = ev\mathcal{B} = e\omega a\mathcal{B}$$

If we assume the charge keeps moving in a circle of the same radius (a vague nod towards quantum mechanics?) it will have a new angular velocity ω',

$$m_\mathrm{e}\omega'^2 a = F - \Delta F$$

so

$$m_\mathrm{e}\omega'^2 a = m_\mathrm{e}\omega^2 a - e\omega a\mathcal{B},$$

or

$$\omega'^2 - \omega^2 = -\frac{e\omega\mathcal{B}}{m_\mathrm{e}}.$$

If the change in frequency is small we have

$$\omega'^2 - \omega^2 \approx 2\omega\Delta\omega,$$

where $\Delta\omega = \omega' - \omega$. Thus

$$\Delta\omega = -\frac{e\mathcal{B}}{2m_\mathrm{e}},$$

the *Larmor frequency*. Substituting back into

$$\mu = IA = e\omega a^2/2.$$

we find a change in magnetic moment

$$\mu = -\frac{e^2 a^2}{4m_\mathrm{e}}\mathcal{B}.$$

Recall that a was the radius of a ring of current perpendicular to the field: if we average over a spherical atom

$$a^2 = \langle x^2\rangle + \langle y^2\rangle = \frac{2}{3}\left[\langle x^2\rangle + \langle y^2\rangle + \langle z^2\rangle\right] = \frac{2}{3}\langle r^2\rangle,$$

so

$$\mu = \frac{e^2\langle r^2\rangle}{6m_\mathrm{e}}\mathcal{B},$$

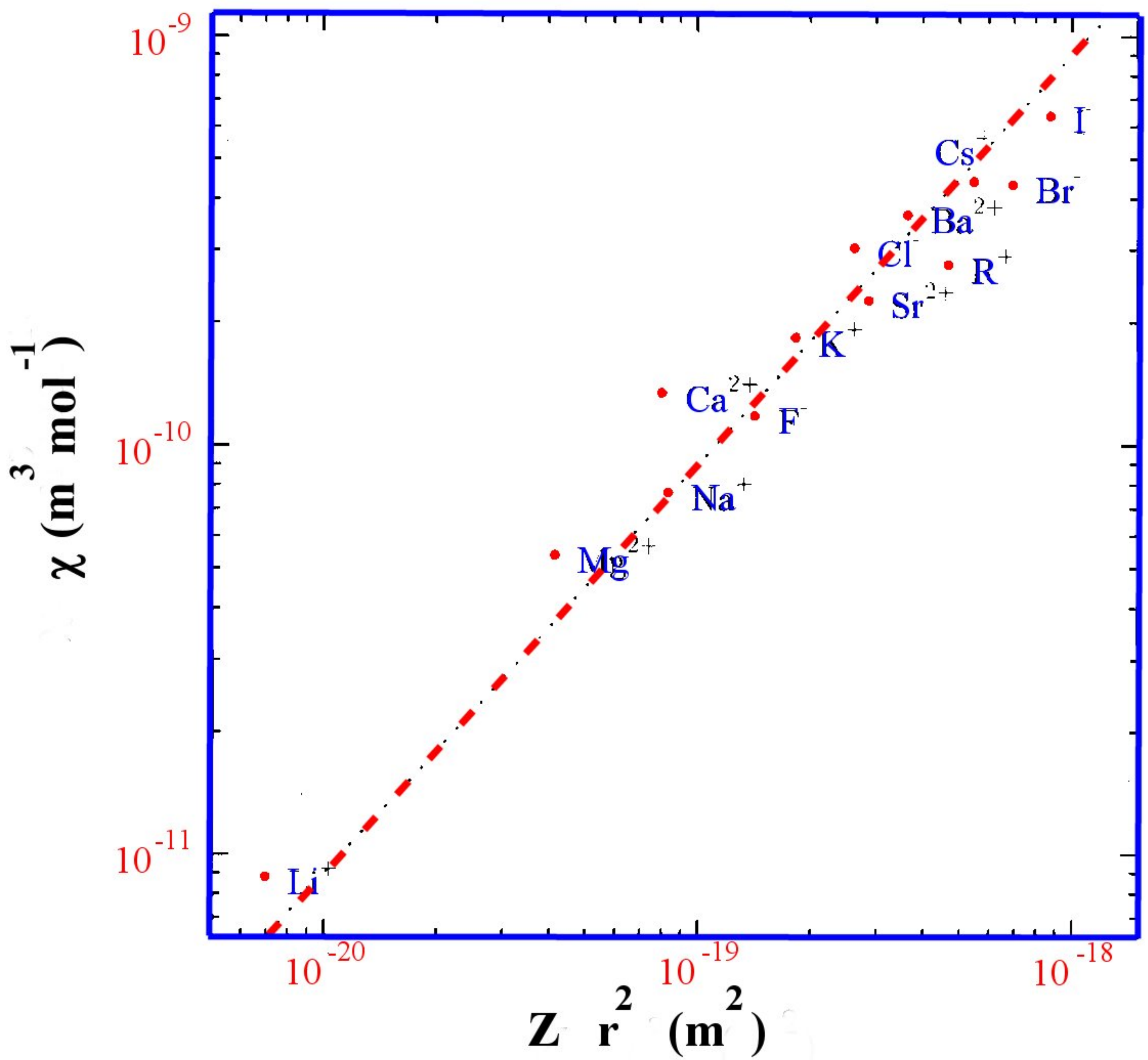
and finally, if we have n atoms per volume, each with p electrons in the outer shells, the magnetisation will be

$$\mathcal{M} = np\mu,$$

and

$$\chi = \frac{\mathcal{M}}{\mathcal{H}} = \mu_0\frac{\mathcal{M}}{\mathcal{B}} = -\frac{\mu_0 npe^2\langle r^2\rangle}{6m_\mathrm{e}}.$$

Values of atomic radius are easily calculated: we can confirm the $p\langle r^2\rangle$ dependence.



10^{-9}
10^{-10}
10^{-11}
χ (m^3 mol^{-1})
10^{-20}
10^{-19}
10^{-18}
Z r^2 (m^2)
I^-
Cs^+
Br^-
Ba^{2+}
Cl^-
R^+
Sr^{2+}
K^+
Ca^{2+}
F^-
Na^+
Mg^{2+}
Li^+

Diamagnetic susceptibility:

- **Negative**
- **Typically** -10^{-6} **to** -10^{-5}
- **Independent of temperature**
- **Always present, even when there are no permanent dipole moments on the atoms.**

10.5 Paramagnetism

Paramagnetism occurs when the material contains permanent magnetic moments. If the magnetic moments do not interact with each other, they will be randomly arranged in the absence of a magnetic field. When a field is applied, there is a balance between the internal energy trying to arrange the moments parallel to the field and entropy trying to randomise them. The magnetic moments arise from electrons, but if we they are localised at atomic sites we can regard them as distinguishable, and use Boltzmann statistics.

10.5.1 Paramagnetism of spin-$\frac{1}{2}$ ions

The spin is either up or down relative to the field, and so the magnetic moment is either $+\mu_{\mathrm{B}}$ or $-\mu_{\mathrm{B}}$, where

$$\mu_{\mathrm{B}} = \frac{e\hbar}{2m_{\mathrm{e}}} = 9.274 \times 10^{-24}\ \mathrm{Am}^2.$$

The corresponding energies in a flux density $\mathcal{B}$ are $-\mu_{\mathrm{B}}\mathcal{B}$ and $\mu_{\mathrm{B}}\mathcal{B}$, so the average magnetic moment per atom is

$$\begin{aligned}\langle\mu\rangle &= \frac{\mu_{\mathrm{B}} e^{\mu_{\mathrm{B}}\mathcal{B}/k_{\mathrm{B}}T} - \mu_{\mathrm{B}} e^{-\mu_{\mathrm{B}}\mathcal{B}/k_{\mathrm{B}}T}}{e^{\mu_{\mathrm{B}}\mathcal{B}/k_{\mathrm{B}}T} + e^{-\mu_{\mathrm{B}}\mathcal{B}/k_{\mathrm{B}}T}} \\ &= \mu_{\mathrm{B}} \tanh\left(\frac{\mu_{\mathrm{B}}\mathcal{B}}{k_{\mathrm{B}}T}\right).\end{aligned}$$

For small z, $\tanh z \approx z$, so for small fields or high temperature

$$\langle\mu\rangle \approx \frac{\mu_{\mathrm{B}}^2\mathcal{B}}{k_{\mathrm{B}}T}.$$

If there are n atoms per volume, then,

$$\chi = \frac{n\mu_0\mu_{\mathrm{B}}^2}{k_{\mathrm{B}}T}.$$

Clearly, though, for low T or large $\mathcal{B}$ the magnetic moment per atom saturates – as it must, as the largest magnetisation possible *saturation magnetisation* has all the spins aligned fully,

$$\mathcal{M}_{\mathrm{S}} = n\mu_{\mathrm{B}}.$$