

# Solid State Physics

## MAGNETISM III

### Lecture 29

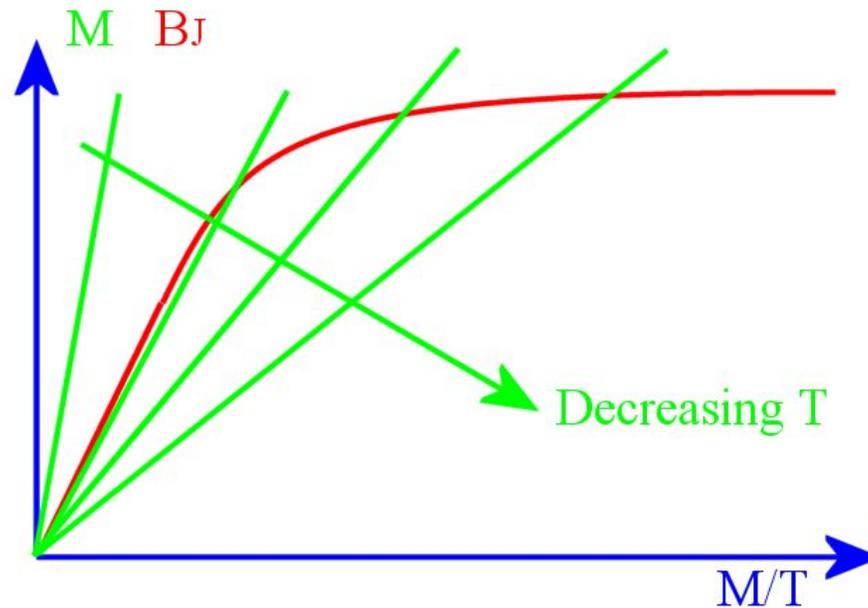
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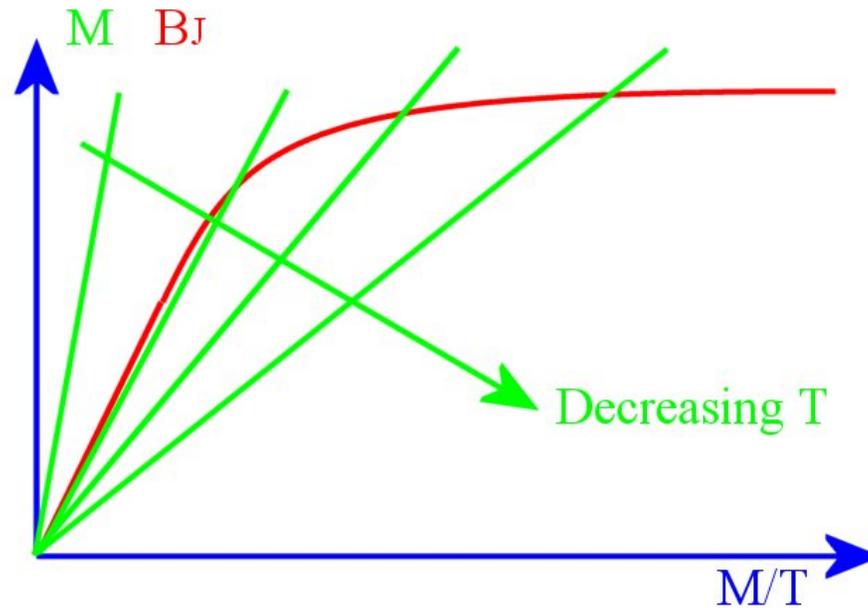
## 10.6.5 Mean field theory of ferromagnetism

Armed with the mean field picture, and a picture of the way  $\mathcal{M}$  depends on  $\mathcal{B}$  through the Brillouin function, we have

$$\frac{\mathcal{M}}{\mathcal{M}_s} = B_J \left( \frac{g_J \mu_B J (\mathcal{B} + \lambda \mathcal{M})}{k_B T} \right). \quad (1)$$

Assume for the moment that  $\mathcal{B} = 0$ . Then we can plot the two sides of equation as functions of  $\mathcal{M}/T$ :

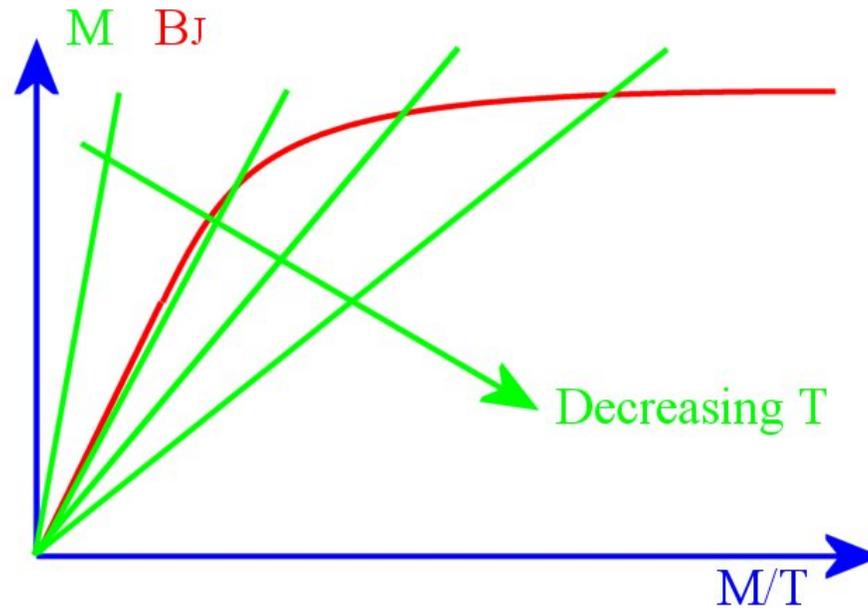




**As  $T$  decreases the straight line  $\mathcal{M}$  gets less steep. Thus for lower  $T$  there is a solution to**

$$\frac{\mathcal{M}}{\mathcal{M}_s} = B_J \left( \frac{g_J \mu_B J \lambda \mathcal{M}}{k_B T} \right)$$

**for finite  $\mathcal{M}$ .**



**Furthermore the shape of  $B_J$ , a convex curve, shows that there is a critical temperature  $T_C$  above which the  $\mathcal{M}$  line is too steep to intersect the  $B_J$  curve except at  $\mathcal{M} = 0$ .**

**For small values of  $\mathcal{M}/T$  we can use Curie's law,**

$$\chi = \frac{\mu_0 n g_J^2 \mu_B^2 J(J+1)}{3k_B T}$$

**and**

$$\chi = \frac{\mathcal{M}}{\mathcal{H}} = \frac{n g_J J \mu_B B_J}{\mathcal{H}}$$

**to deduce**

$$B_J \left( \frac{g_J \mu_B J \mathcal{B}}{k_B T} \right) \approx \frac{g_J \mu_B (J+1) \mathcal{B}}{3k_B T}.$$

**In terms of  $x = \mathcal{M}/T$ , the straight line is**

$$\frac{\mathcal{M}}{\mathcal{M}_s} = \frac{T x}{\mathcal{M}_s}$$

**and the approximation to the Brillouin function is (putting  $\lambda \mathcal{M}$  for  $\mathcal{B}$ )**

$$B_J \approx \lambda \mathcal{M} \frac{g_J \mu_B (J+1)}{3k_B T} = \lambda \frac{g_J \mu_B (J+1)}{3k_B} x.$$

**Equating the gradients with respect to  $x$ ,**

$$\frac{T_C}{\mathcal{M}_s} = \lambda \frac{g_J \mu_B (J+1)}{3k_B},$$

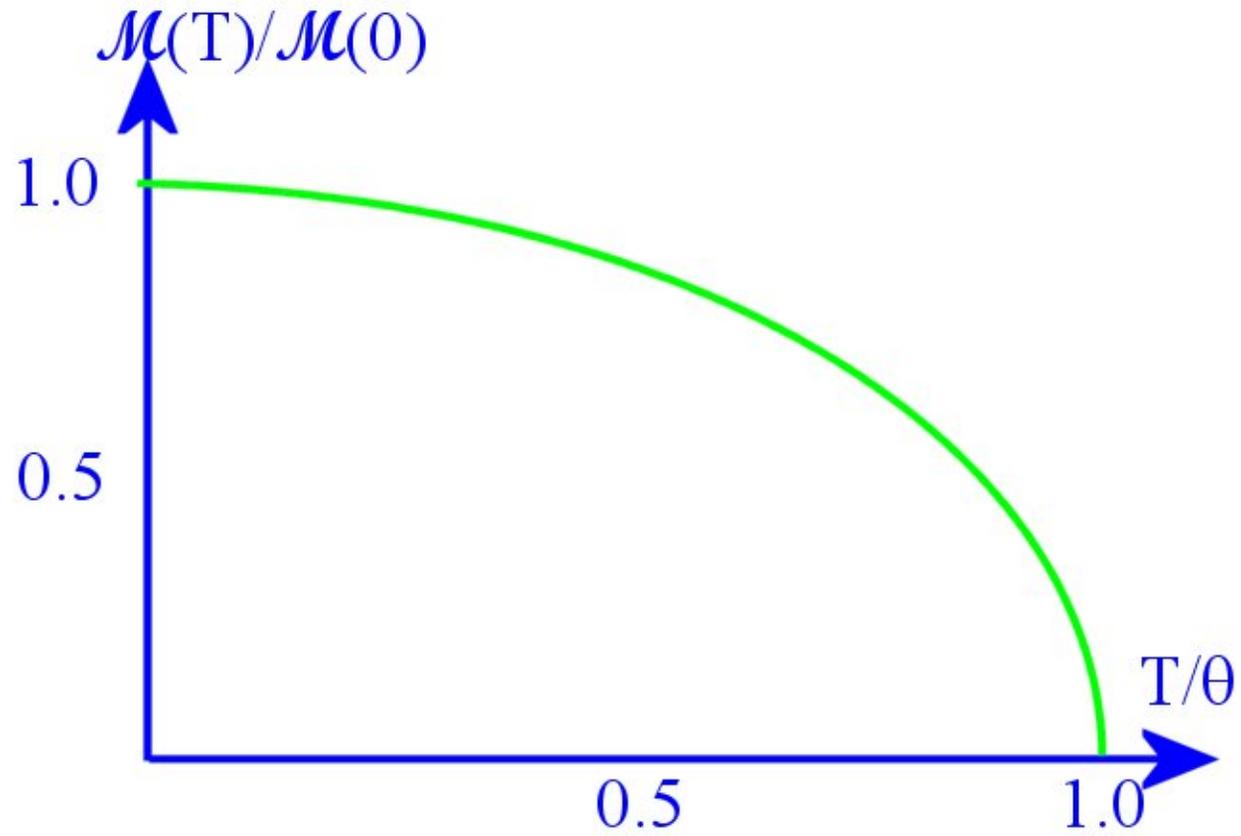
**or**

$$T_C = \lambda \frac{g_J \mu_B (J + 1) \mathcal{M}_s}{3k_B}$$
$$= \frac{\lambda n g_J^2 \mu_B^2 J(J + 1)}{3k_B}.$$

**The critical temperature  $T_C$  is the *Curie temperature* – often denoted by  $\theta$ . Some ferromagnetic materials**

<b>Material</b>	<b><math>T_C</math> (K)</b>	<b><math>\mu_B</math> per formula unit</b>
<b>Fe</b>	<b>1043</b>	<b>2.22</b>
<b>Co</b>	<b>1394</b>	<b>1.715</b>
<b>Ni</b>	<b>631</b>	<b>0.605</b>
<b>Gd</b>	<b>289</b>	<b>7.5</b>
<b>MnSb</b>	<b>587</b>	<b>3.5</b>
<b>EuO</b>	<b>70</b>	<b>6.9</b>
<b>EuS</b>	<b>16.6</b>	<b>6.9</b>

**Below  $T_C$  the spontaneous magnetisation varies with temperature.**



## 10.6.6 Paramagnetic regime

Above the Curie temperature, if we apply a magnetic field, we have

$$B_J = \frac{\mathcal{M}}{\mathcal{M}_s} \approx (\mathcal{B} + \lambda\mathcal{M}) \frac{g_J \mu_B (J + 1)}{3k_B T}$$

which can be rearranged to give

$$\mathcal{M} = \frac{\frac{\mathcal{M}_s \mathcal{B} g_J (J+1) \mu_B}{3k_B}}{T - \frac{\lambda \mathcal{M}_s g_J (J+1) \mu_B}{3k_B}},$$

and with  $\mathcal{M}_s = n g_J J \mu_B$

$$\mathcal{M} = \frac{\frac{n \mathcal{B} g_J^2 J (J+1) \mu_B^2}{3k_B}}{T - \frac{\lambda n g_J^2 J (J+1) \mu_B^2}{3k_B}},$$

or

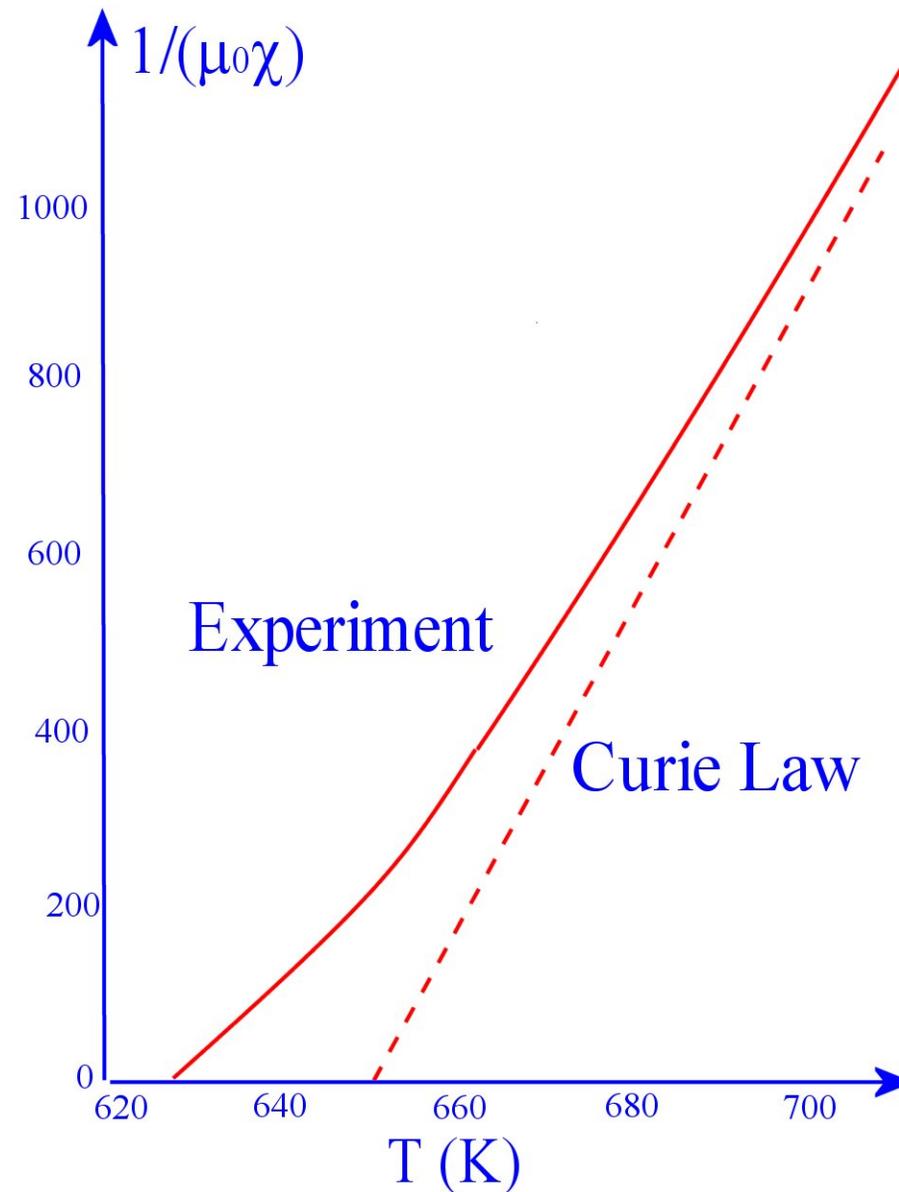
$$\mathcal{M} = \frac{\frac{n \mathcal{B} g_J^2 J (J+1) \mu_B^2}{3k_B}}{T - T_C}.$$

**This gives a susceptibility**

$$\chi \propto \frac{1}{T - T_C},$$

**the *Curie-Weiss law*.**

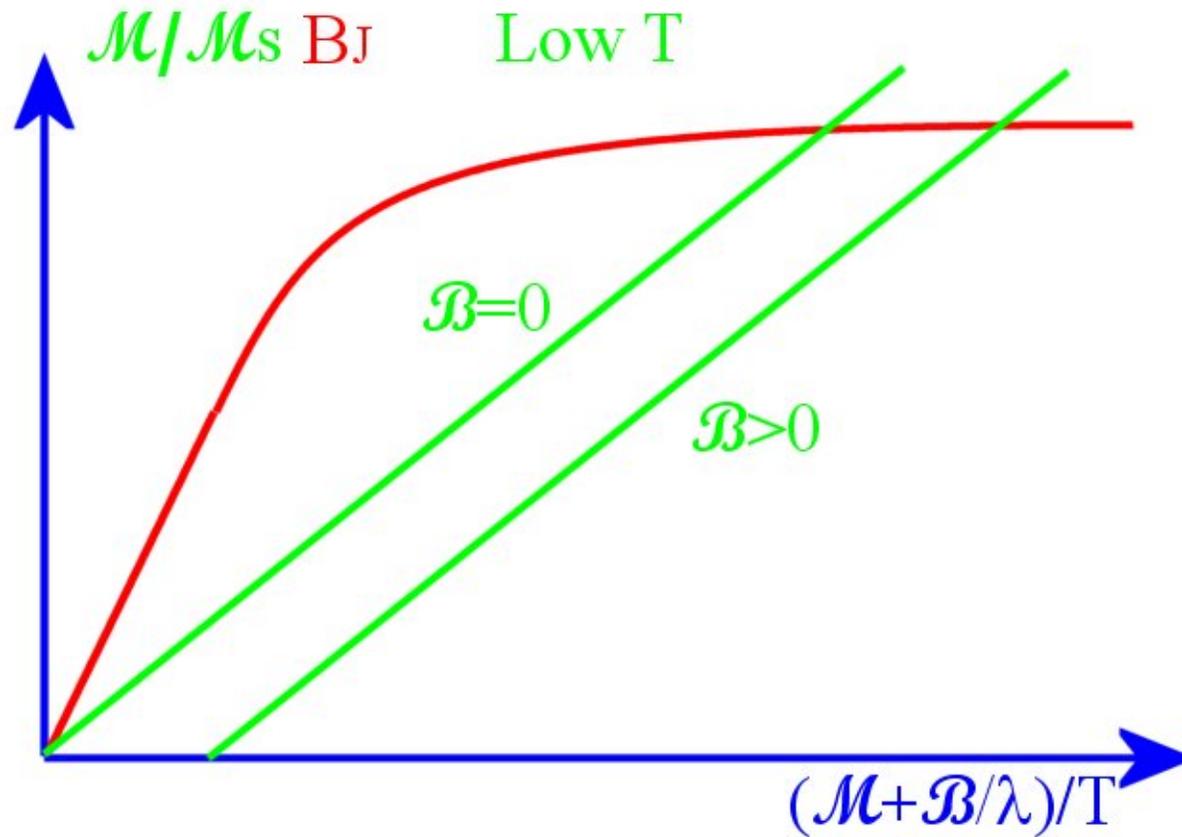
## The Curie-Weiss law works quite well at high $T$



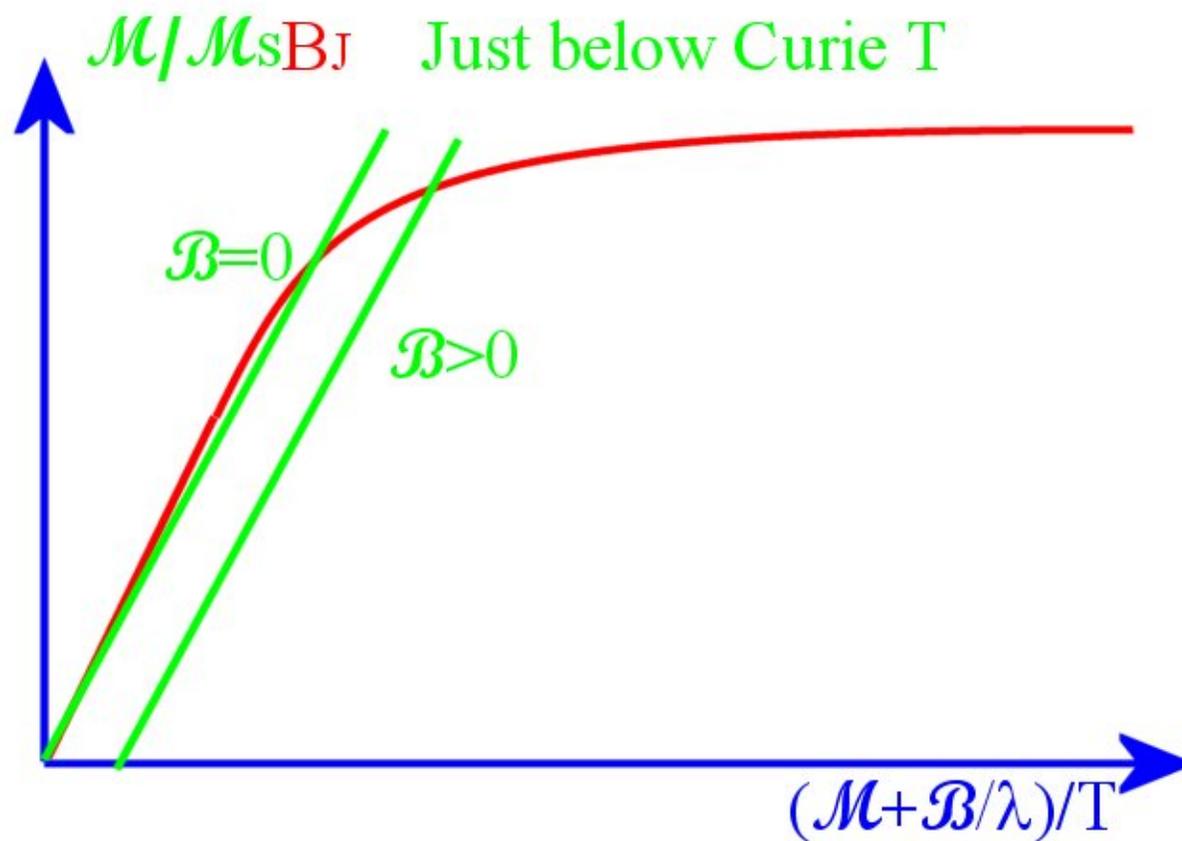
but breaks down near the Curie temperature  $T_C$  or  $\theta$ , where the mean field approximation fails.

## 10.6.7 Effect of magnetic field on ferromagnet

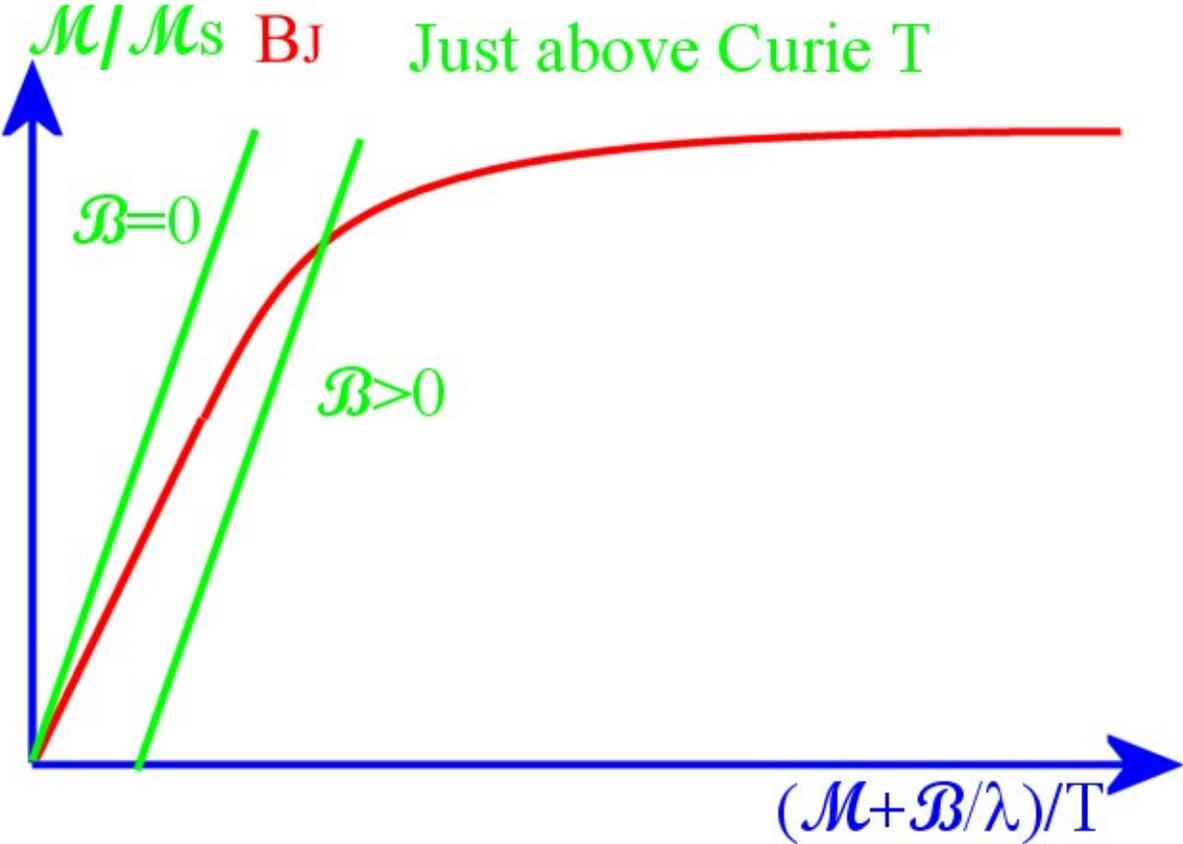
At low temperatures, the magnetisation is nearly saturated, so a  $\mathcal{B}$  field has little effect:



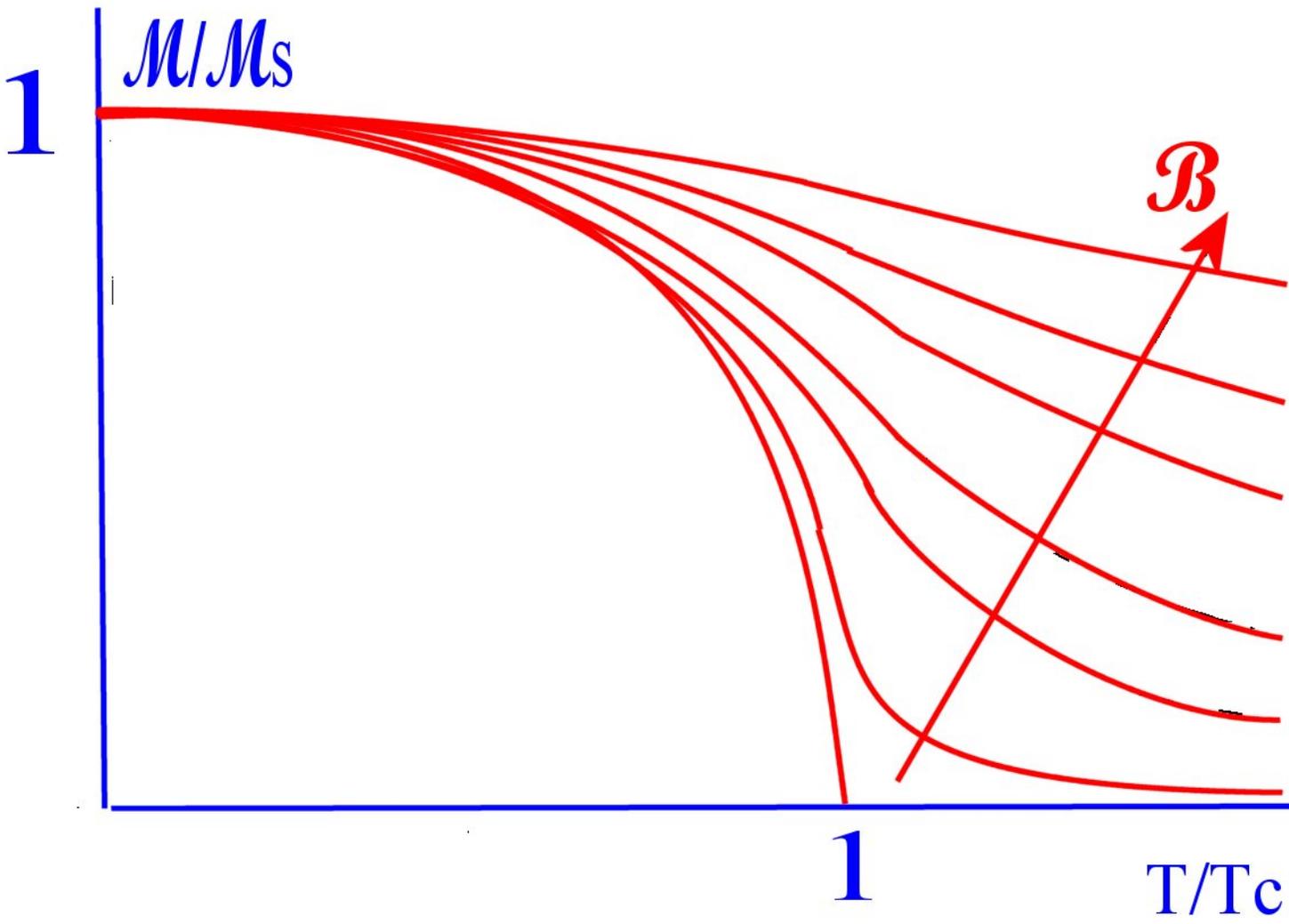
As we increase the temperature, we reach a regime where the field has a large effect on the magnetisation:



At high temperature we are in the Curie-Weiss regime than we described above:

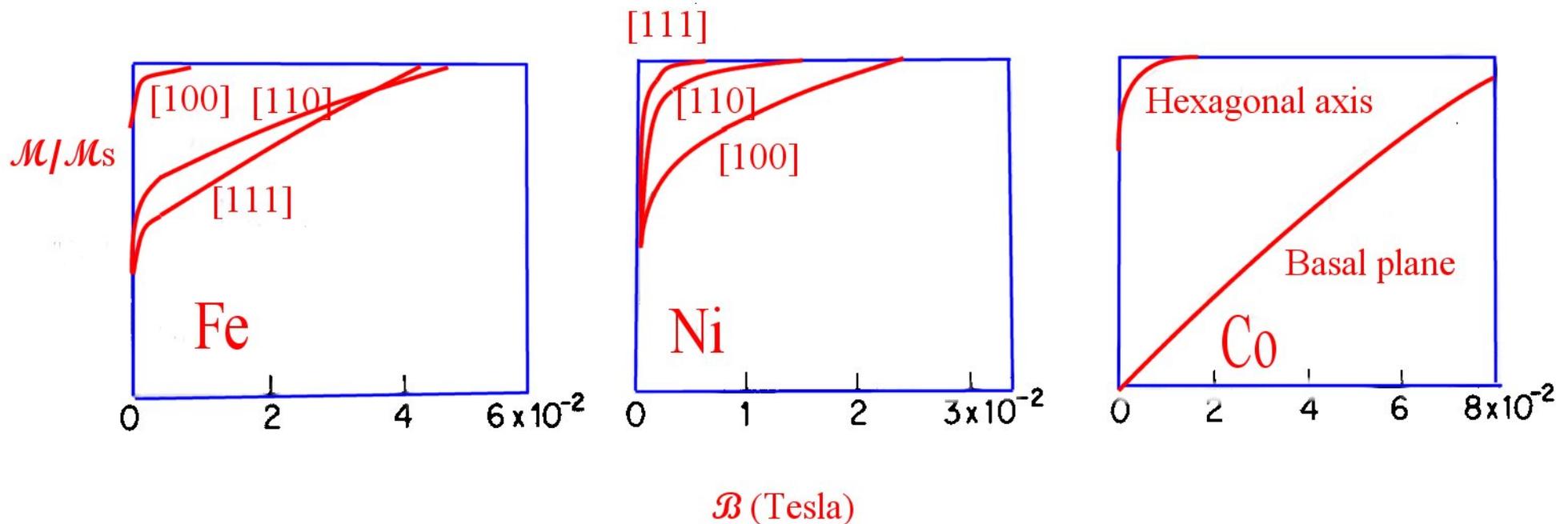


Overall, then, the effect of a field is:



## 10.6.8 Anisotropy in magnetic systems

The quenching of orbital angular momentum in a crystal is one effect of the *crystal field* – the electrostatic potential variation in the solid. But as spin-orbit coupling links the spins to the spatial variation of the wavefunctions, the spins tend to align more readily along certain directions in the crystal – the *easy directions of magnetisation*.



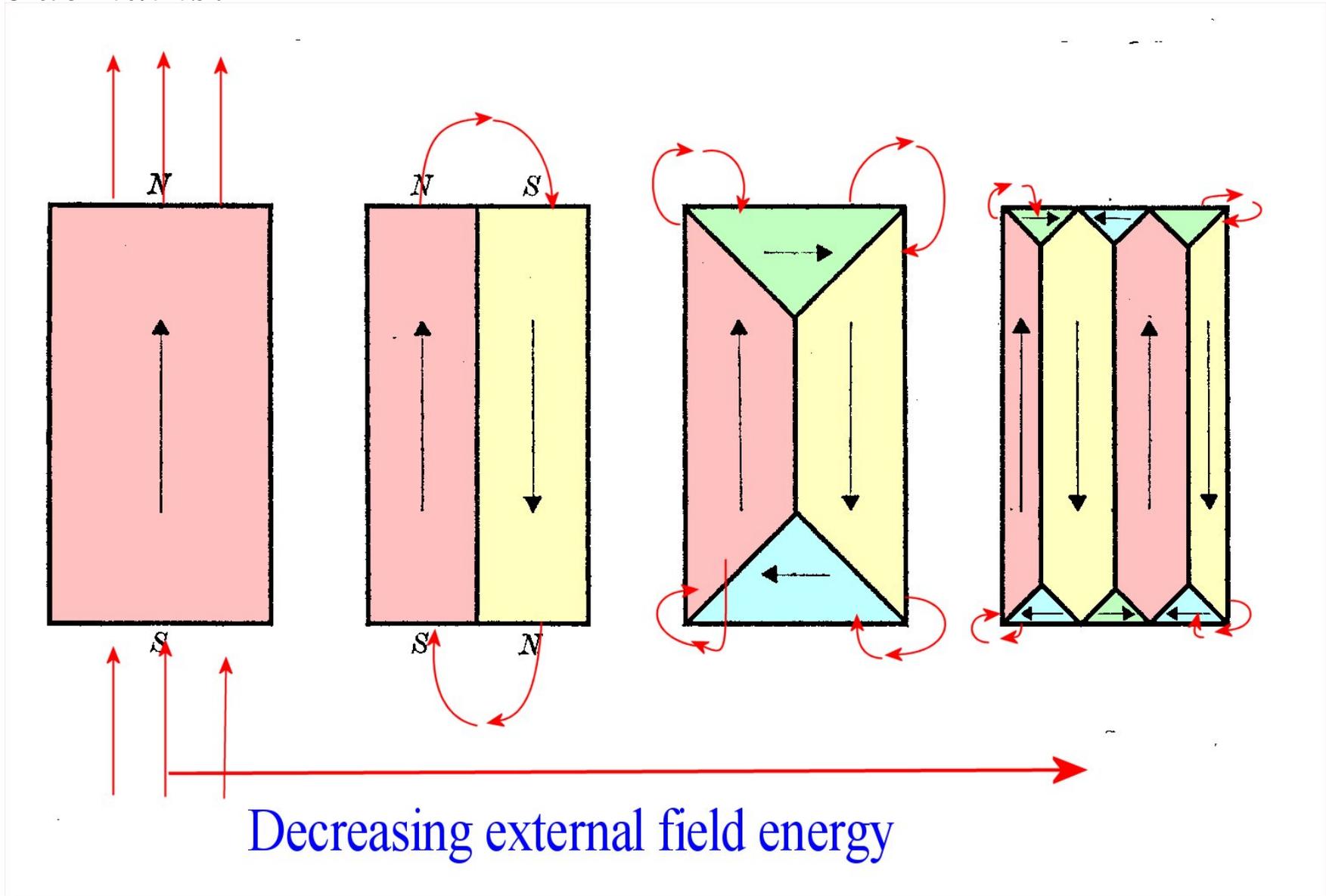
## 10.7 Magnetic domains

In general, a lump of ferromagnetic material will not have a net magnetic moment – despite the fact that internally the spins tend to align parallel to one another.

### 10.7.1 Magnetic field energy

The total energy of a ferromagnetic material has two components – the internal energy (including the exchange energy) tending to align spins and the energy  $\int \mathcal{B} \cdot \mathcal{H} dV$  in the field outside it.

The external field energy can be decreased by dividing the material into *domains*.



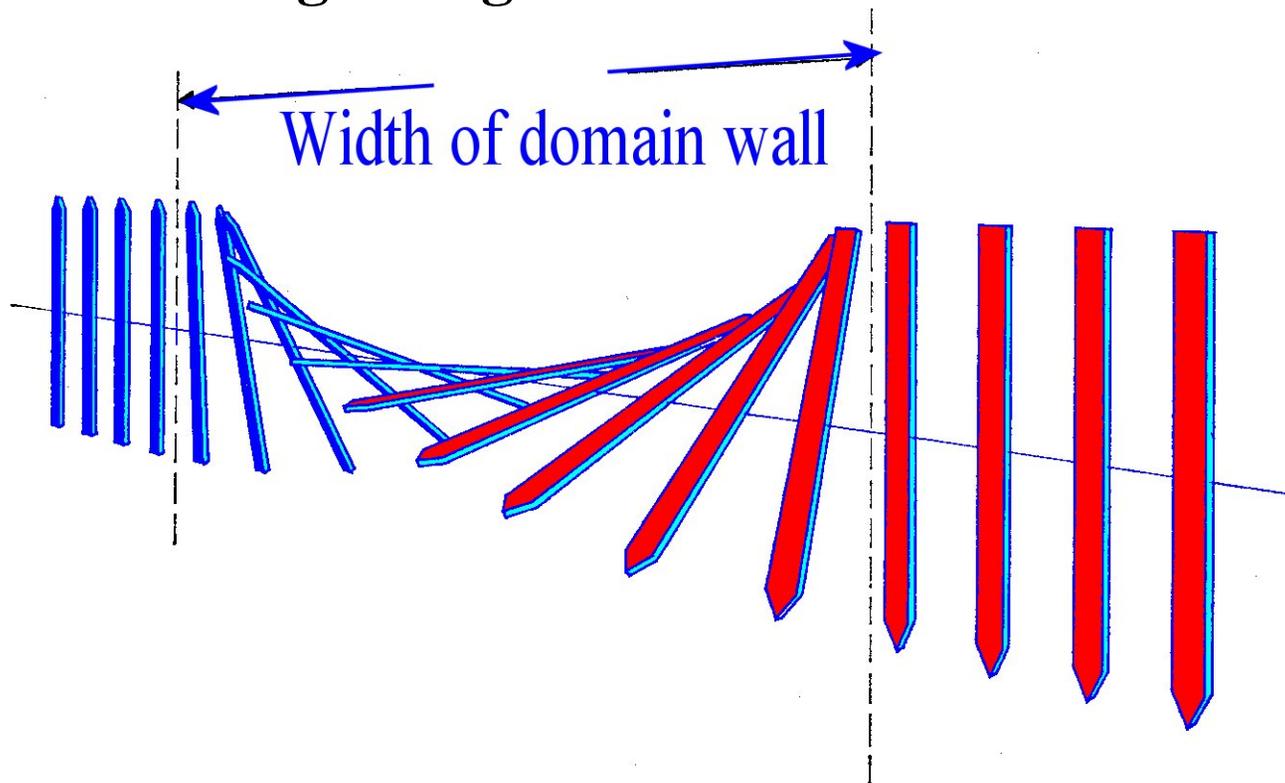
The internal energy is increased because not all the spins are now aligned parallel to one another.

## 10.7.2 Domain walls

What is the structure of the region between two domains (called a *domain wall* or a *Bloch wall*)? The spins do not suddenly flip: a gradual change of orientation costs less energy because if successive spins are misaligned by  $\delta\theta$  the change in energy is only

$$\delta E = 2JS^2(1 - \cos(\delta\theta)),$$

where  $J$  is the exchange integral.



**For small  $\delta\theta$ , expanding the cosine,**

$$\delta E = 2JS^2 \frac{1}{2} (\delta\theta)^2$$

**and if we extend the change in spin direction (total angle change of  $\pi$ ) over  $N$  spins,  $\delta\theta = \pi/N$ , and there are  $N$  such changes of energy  $\delta E$ , so the total energy change is**

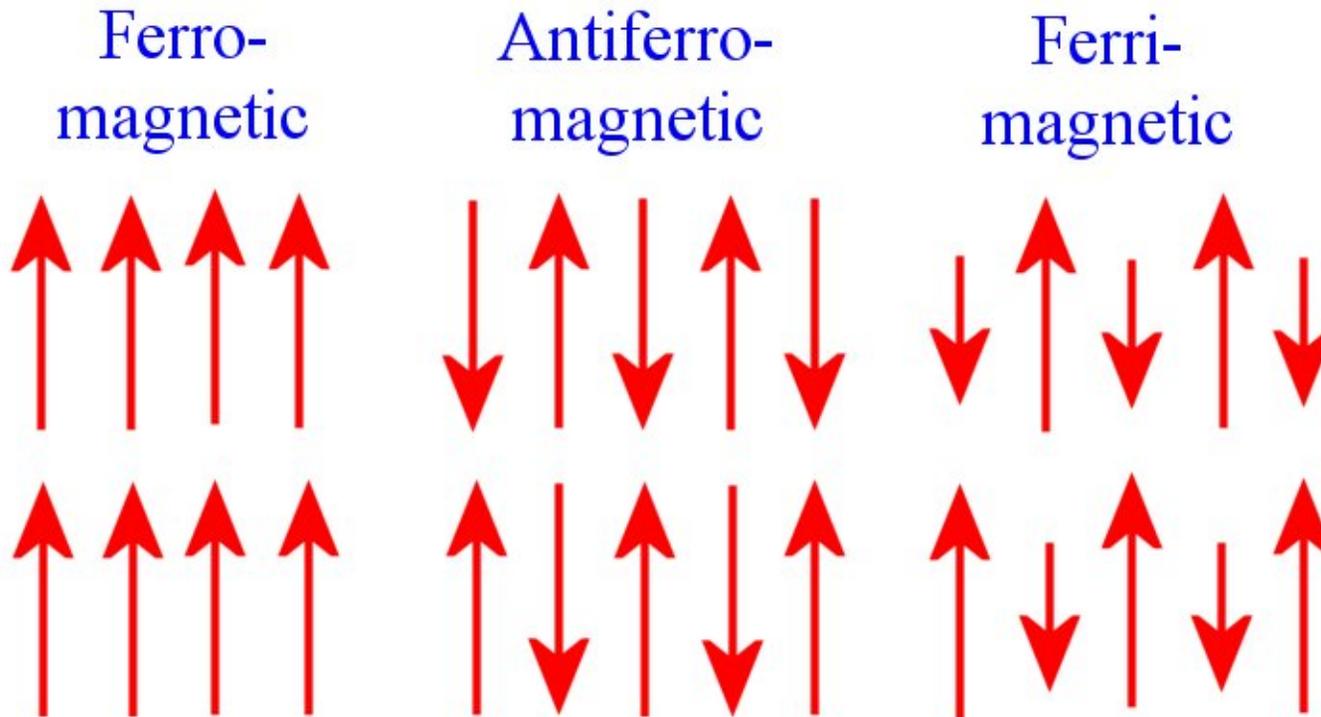
$$\Delta E = JS^2 \frac{\pi^2}{N}.$$

**This favours wide walls, but then there are more spins aligned away from easy directions, providing a balance. Bloch walls are typically about 100 atoms thick.**

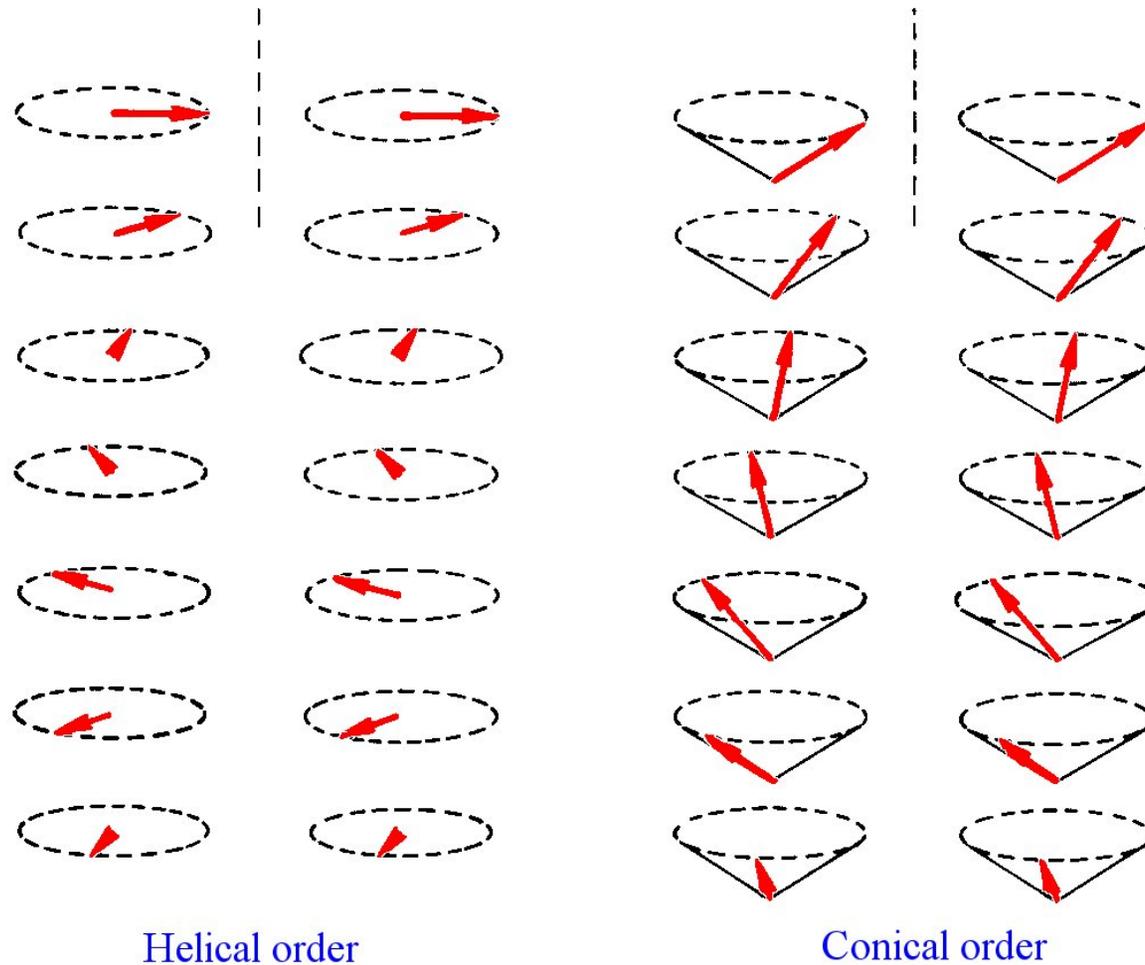
**In very small particles, the reduction in field energy is too small to balance the domain wall energy – small particles stay as single domains – *superparamagnets*. Small magnetic particles are found in some bacteria (*magnetotactic bacteria*) which use the angle of dip of the Earth's magnetic field to direct them to food.**

## 10.8 Other types of magnetic ordering

The three easiest types of magnetic ordering to visualise are ferromagnetic (all spins aligned parallel), antiferromagnetic (alternating spins of equal size), and ferrimagnetic (alternating spins of different size, leading to nett magnetic moment).



As the exchange integral  $J$  can have complicated dependence on direction, other orderings are possible, for example:



**Helical ordering (spins parallel within planes, but direction changing from plane to plane) – e.g. Dy between 90 and 180 K.**

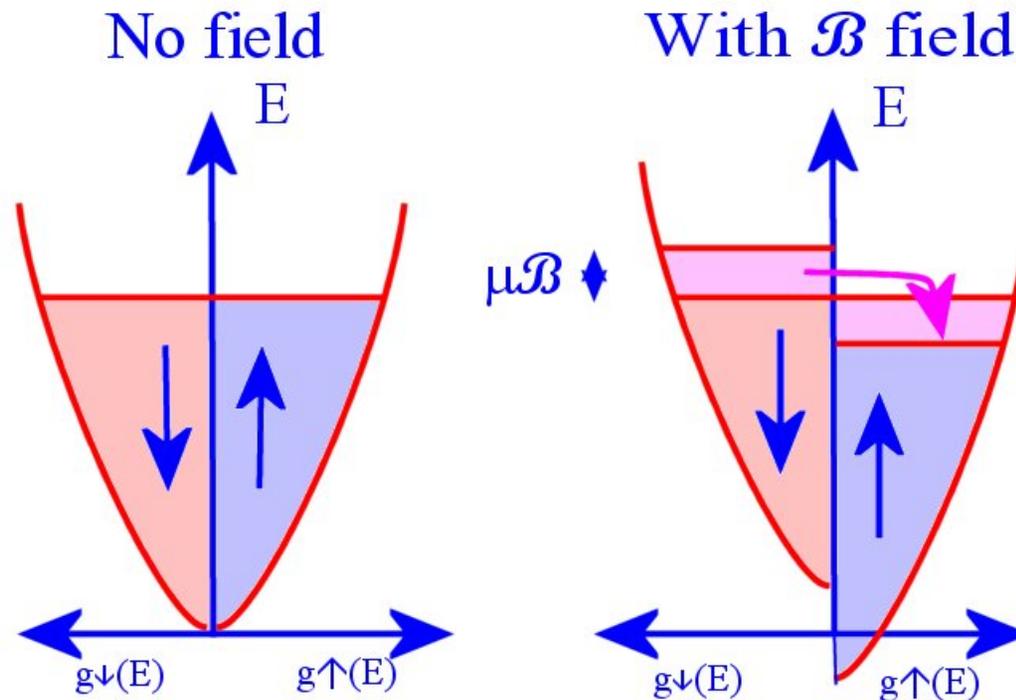
**Conical ordering – e.g. Eu below 50 K.**

**Polarised neutron scattering reveals these structures.**

## 10.9 Magnetic properties of metals

### 10.9.1 Free electron paramagnetism

In a metal, the free electrons have spins, which can align in a field. As the electrons form a degenerate Fermi gas, the Boltzmann statistics we have used so far are inappropriate.



The field  $\mathcal{B}$  will shift the energy levels by  $\pm\mu_B\mathcal{B}$ .

Thus the number of *extra* electrons per unit volume with spin up will be

$$\Delta n_{\uparrow} = \frac{1}{2}g(E_F)\mu_B\mathcal{B}$$

and there is a corresponding change in the number with spin down,

$$\Delta n_{\downarrow} = -\frac{1}{2}g(E_F)\mu_B\mathcal{B}.$$

The magnetisation is therefore

$$\mathcal{M} = \mu_B(n_{\uparrow} - n_{\downarrow}) = g(E_F)\mu_B^2\mathcal{B},$$

giving a susceptibility

$$\chi = \frac{\mathcal{M}}{\mathcal{H}} = \mu_0\mu_B^2g(E_F) = \frac{3n\mu_0\mu_B^2}{2E_F}.$$

This is a *temperature-independent* paramagnetism, typically of order  $10^{-6}$ . The free electrons also have a diamagnetic susceptibility, about  $-\frac{1}{3}$  of the paramagnetic  $\chi$ .

## 10.9.2 Ferromagnetic metals

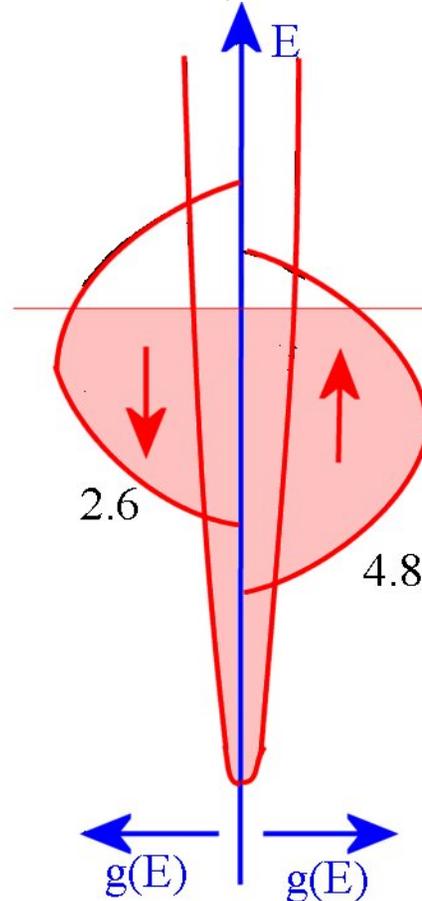
If we look at the periodic table we find that the ferromagnetic elements are metals.

IA											VIII						
H	IIA											III B	IV B	V B	V I B	V II B	He
Li	Be											B	C	N	O	F	Ne
Na	Mg	III A	IV A	V A	V I A	V II A	V III A	IB	IIB	Al	Si	P	S	Cl	Ar		
K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr
Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe
Cs	Ba	La	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn
Fr	Ra	Ac															
		Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu		
		Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es						

Antiferromagnetic  
 Ferromagnetic

This causes some complication in the magnetic properties – treated in a simplified way by *Stoner theory*.

**The exchange interaction splits the narrow d bands: the wide free-electron-like s bands are relatively unaffected.**



**The Fermi surface is determined by the total number of electrons: this can lead to apparently non-integer values of the magnetic moment per atom (e.g. 2.2 in Fe, 0.6 in Ni).**