SUPERCONDUCTIVITY II

Lecture 31

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11.3 Phenomenological theories

By showing that

$$\frac{\mathrm{d}\mathcal{B}}{\mathrm{d}t} = Ae^{-x/\lambda}$$

we have still not established a difference between a perfect conductor and a superconductor – it is only the *rate of change* of $\mathcal{B}$ that we have shown to decay within the material. The brothers F. and H. London suggested, in 1935, that a superconductor should obey the equation

$$\nabla \times \mathcal{J} = -\frac{nq^2}{m}\mathcal{B} \tag{1}$$

in addition to the equation for non-scattered carriers

$$\frac{\mathrm{d}\mathcal{J}}{\mathrm{d}t} = \frac{nq^2}{m}\mathcal{E}. \tag{2}$$

Then, as before, take the curl of both sides of Maxwell's equation (with no displacement currents) $\nabla \times \mathcal{H} = \mathcal{J}$.

$$\nabla \times \mathcal{H} = \mathcal{J}$$

gives

$$\nabla \times \nabla \times \mathcal{B} = \mu_0 \nabla \times \mathcal{J}$$

whence, as $\nabla \times \nabla \times = \nabla(\nabla .) - \nabla^2$**, we have**

$$-\nabla^2 \mathcal{B} = \mu_0 \nabla \times \mathcal{J}$$

and, from equation 1 $\nabla \times \mathcal{J} = -(nq^2/m)\mathcal{B}$**,**

$$\nabla^2 \mathcal{B} = \frac{\mu_0 n q^2}{m} \mathcal{B}.$$

Then, with

$$\lambda = \sqrt{\frac{m}{\mu_0 n q^2}},$$

we have

$$\mathcal{B}(x) = \mathcal{B}(0) e^{-x/\lambda}.$$

Similarly

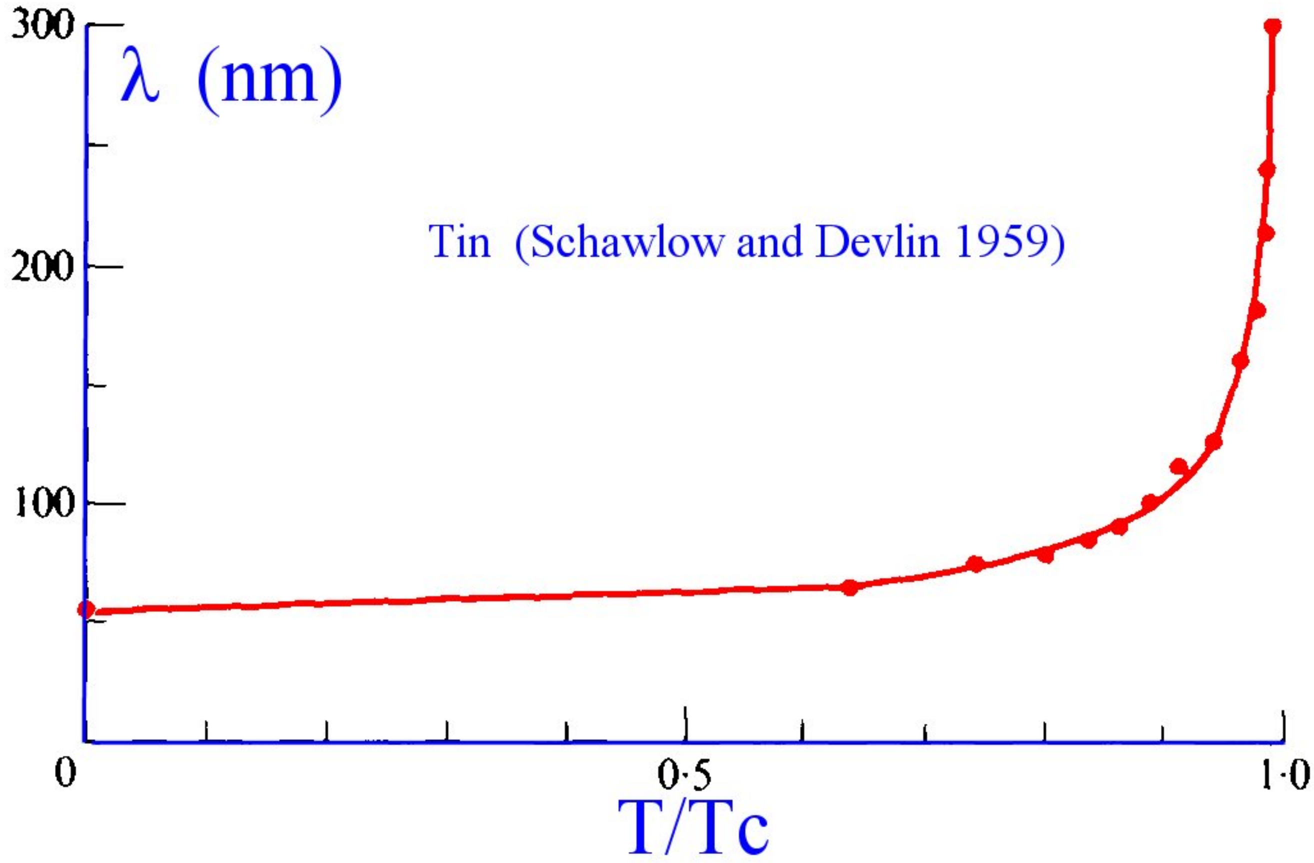
$$\mathcal{J}(x) = \mathcal{J}(0) e^{-x/\lambda}.$$

Now we have a decay of the *static* field. Note that the London equations do not allow a uniform non-zero field inside the material: if

the field inside is constant it must be zero. If we assume that *all* the electrons are involved in the unscattered current, we find $\lambda \approx 10^{-8} \text{ to } 10^{-7}\,\mathrm{m}$, the London penetration depth.

11.3.1 Measurement of penetration depth

If all flux were excluded from a superconductor, there would be no flux linkage between two coils wound on a superconducting core. As there is flux throughout the penetration region, λ can be measured by measuring the mutual inductance of the coils.



Many experiments show that λ varies with temperature, to a good approximation, as

$$\lambda(T) = \frac{\lambda(0)}{\sqrt{1 - \left(\frac{T}{T_\mathrm{C}}\right)^4}}.$$

Recalling that

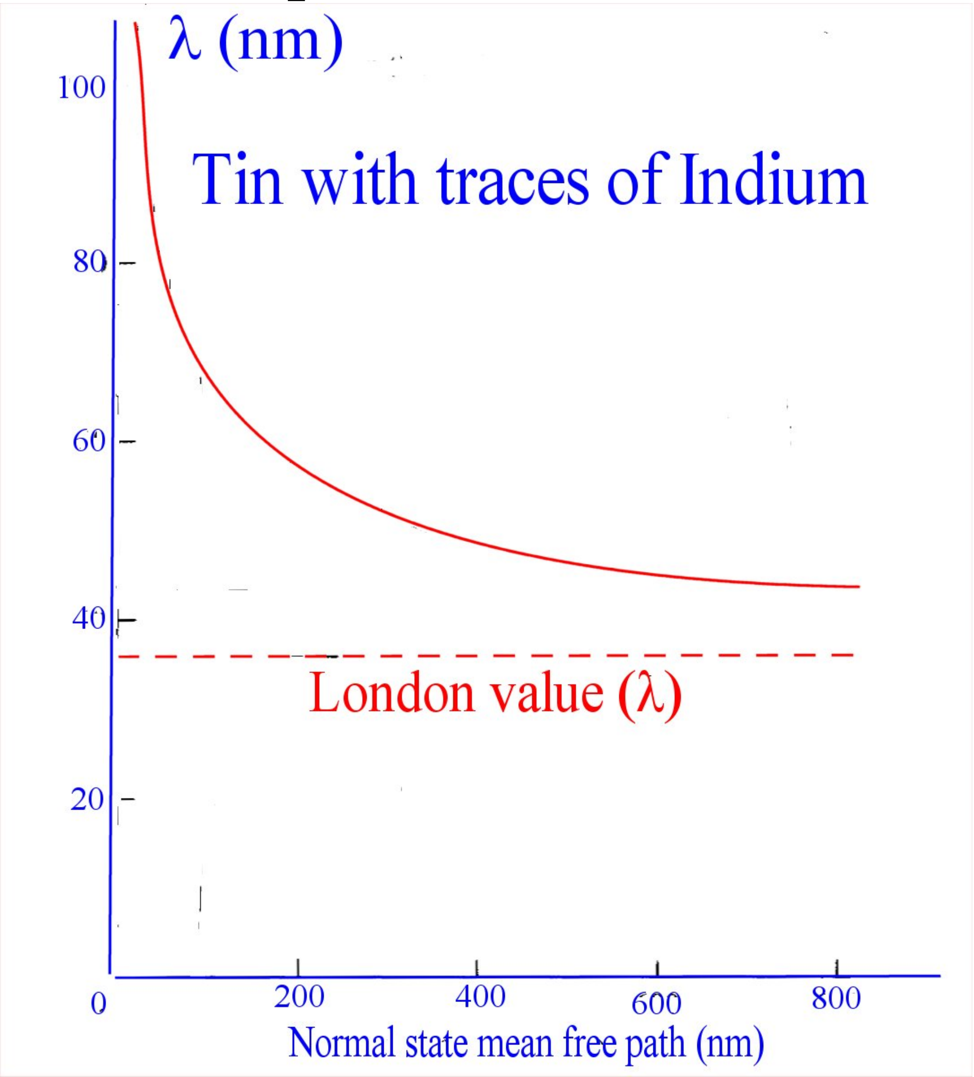
$$\lambda = \sqrt{\frac{m}{\mu_0 n q^2}},$$

this suggests that the number of unscattering carriers varies as

$$1 - \left(\frac{T}{T_\mathrm{C}}\right)^4.$$

11.4 Coherence

Unfortunately, experiment showed that the penetration depth does not just depend on T, but also on impurities. Penetration depth and normal electron mean free path are related.

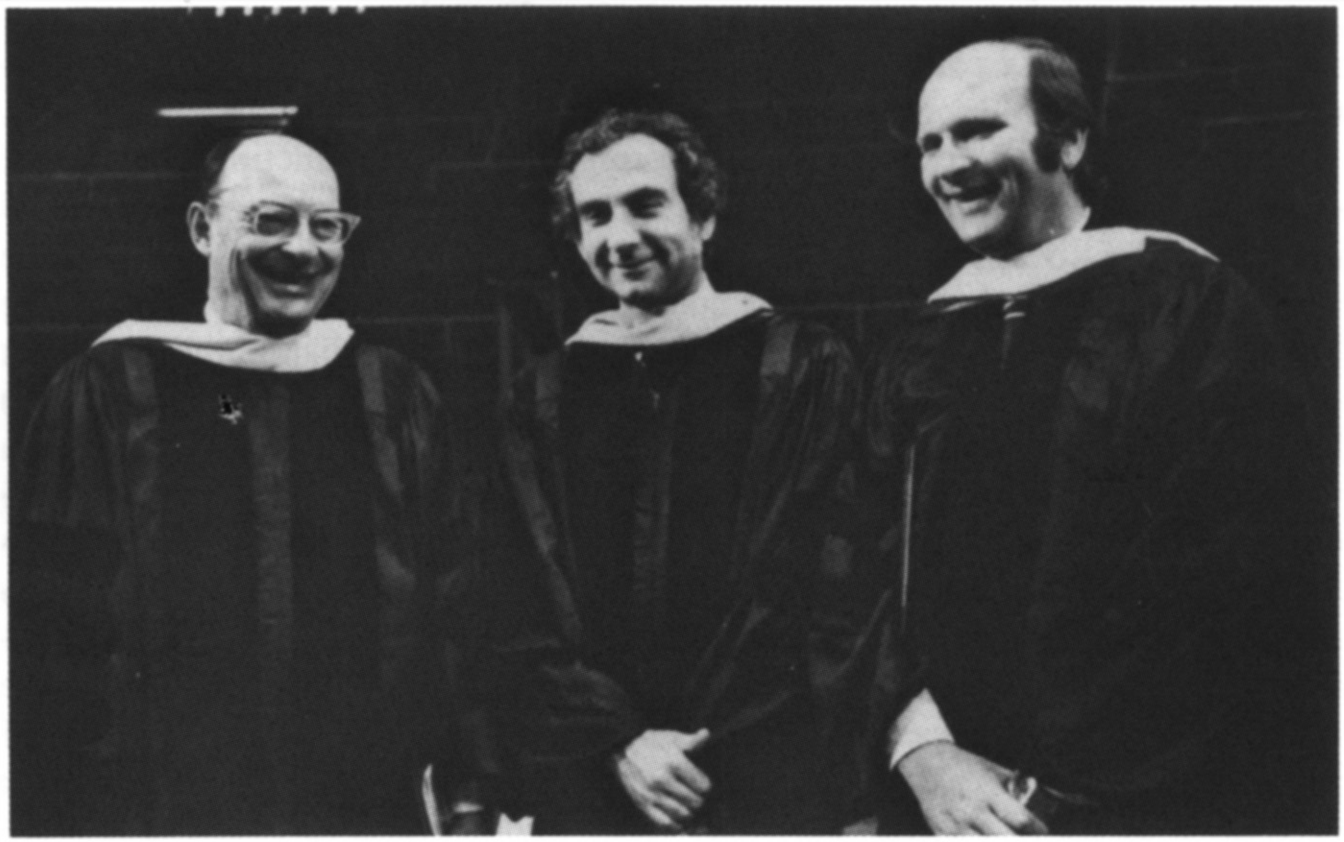


Pippard suggested that the superconducting state was one of *long-range order* over some *coherence length*, ξ. Evidence for this includes:

- **The sharpness of the superconducting transition – if electrons were individually going into some new state there would be statistical fluctuations giving broader transitions.**
- **The penetration depth dependence on mean free path – assume that we can only determine the *average* superconducting current over a volume ξ^3:**
  - ***long mean free path and large ξ*: averaging gives non-local relationship between $\mathcal{B}$ and $\mathcal{J}$.**
  - ***impure materials* with $\xi \approx \mathrm{mean\ free\ path}$ have greatly increased λ**
  - ***small ξ* recovers original local model for λ.**

11.5 Microscopic model

In 1957 Bardeen, Cooper and Schrieffer put together the clues to provide the BCS theory of superconductivity[1].



John Bardeen (b. 1908), Leon N. Cooper (b. 1930), and John Robert Schrieffer (b. 1931). (AIP Niels Bohr Library)

[1]John Bardeen was the first person to receive two Nobel prizes in the same field. He shared the 1956 prize for physics with William Shockley and Walter Brattain for the discovery of the transistor effect, and the 1972 prize with Leon Cooper and John Schrieffer for their theory of superconductivity

Cooper took the first step in 1956 by showing that if two electrons are added to the ground state of the free electron gas (filled states up to E_{F} they will form a bound state ($E < 2E_{\mathrm{F}}$) if there is an attractive potential *however small* between them. If there is an attractive interaction of strength V between electrons in an energy range $\hbar\omega$ above E_{F}, then their energy will be reduced by

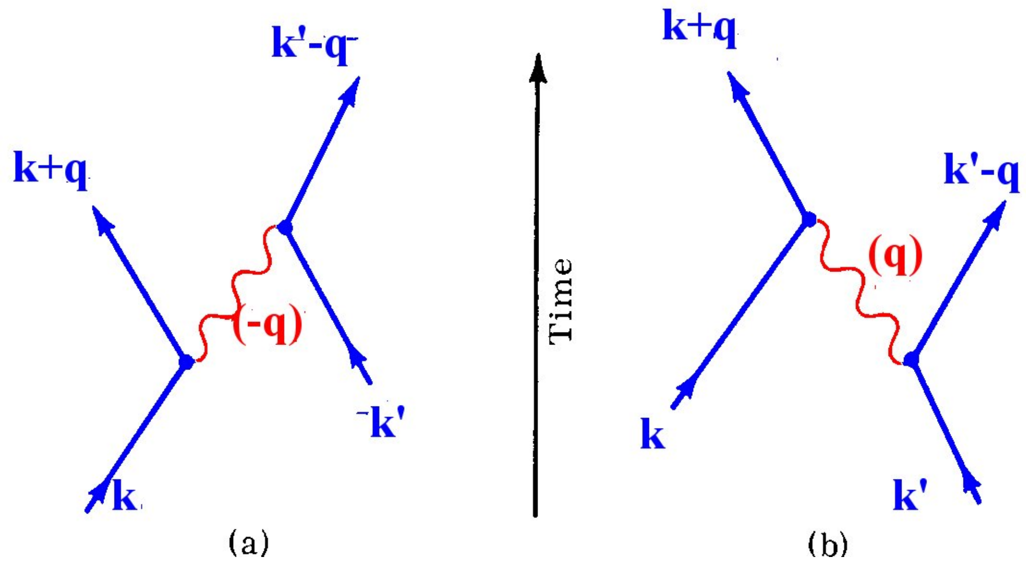
$$\Delta = -2\hbar\omega e^{-2/(g(E_{\mathrm{F}})V)},$$

provided that $g(E_{\mathrm{F}})V$ is small. Δ is typically about 1 meV. The V in the denominator of the exponential shows that any attempts to predict superconductivity using perturbation theory were doomed to failure. The bound pair (Cooper pair) has opposite values of k and opposite spins.

Cooper's discovery could be linked with Fröhlich's (1950) suggestion that

- **an electron moving through the positively charged ion cores will displace them slightly from their normal positions**
- **this local increase in positive charge density attracts another electron.**

Alternative explanation in terms of *virtual phonons*.



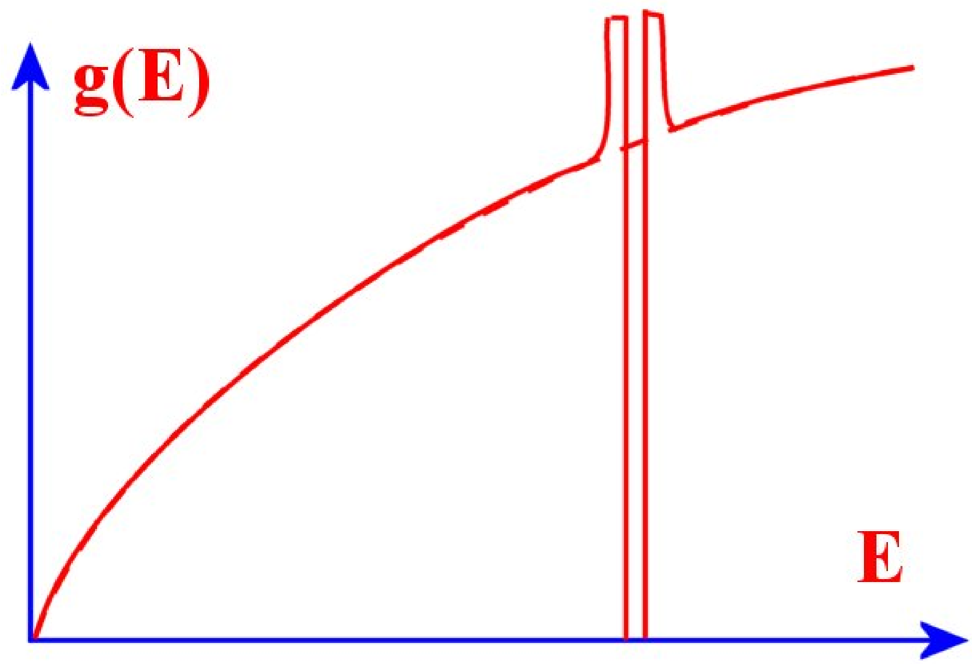
- **an electron with** k **emits a phonon** q
- **if the phonon is rapidly absorbed by another electron in time** Δt **the uncertainty relation** $\Delta E \Delta t \geq \hbar$ **lets us 'borrow' energy** ΔE
- **the phonon is absorbed by another electron**
- **this may change the energy of the electrons, if**

$$|\mathbf{k}|^2 + |\mathbf{k}'|^2 \neq |\mathbf{k}+\mathbf{q}|^2 + |\mathbf{k}'-\mathbf{q}|^2.$$

- **as phonon frequencies** $\omega \propto \sqrt{\kappa/M}$ **for force constant** κ **and mass** M **this is consistent with the isotope effect**

11.5.1 The energy gap

The effect of the interaction is to ensure that within $\triangle$ of the Fermi surface there are no occupied states. The density of states immediately above and below the gap is increased correspondingly.



The gap is $2\triangle$ wide. The Fermi energy is in the middle of the gap. An energy $2\triangle$ will break up a pair and create two 'normal' electrons. The pairs have many of the properties of bosons.

11.5.2 The wavefunction

The wavefunction for the paired electrons corresponds to electrons with energies within Δ of E_{F}. Now

$$\Delta = \delta E = \delta\left(\frac{\hbar^2 k^2}{2m}\right) \approx \left(\frac{\hbar k_{\mathrm{F}}}{m}\right)\hbar\delta k.$$

If we assume that the spread of the wavefunction is determined by the uncertainty relation,

$$\xi\delta(\hbar k) \approx \hbar,$$

we find

$$\xi \approx \frac{1}{\delta k} \approx \frac{\hbar k_{\mathrm{F}}}{m\Delta} \approx \frac{1}{k_{\mathrm{F}}}\ \frac{E_{\mathrm{F}}}{\Delta},$$

and putting in typical values of $E_{\mathrm{F}}/\Delta \approx 10^3$, $k_{\mathrm{F}} \approx 10^{10}\ \mathrm{m}^{-1}$, $\xi \approx 10^{-7}\ \mathrm{m}$. Note that ξ can be large compared with the London penetration depth.

Within the coherence length there are millions of Cooper pairs – energy favours their having the same phase. This is the ordering. Often write the superconducting wavefunction as

$$\psi(\mathbf{r}) = \sqrt{n_s(\mathbf{r})}e^{i\theta(\mathbf{r})} :$$

$n_s(\mathrm{r})$ **is the density of pairs,** $\theta(\mathrm{r})$ **describes a spatially varying phase.**

Minimising the free energy one finds the critical temperature is given by

$$k_{\rm B}T_{\rm C} = 1.14\hbar\omega e^{-2/(g(E_{\rm F})V)},$$

so

$$2\Delta = 3.52k_{\rm B}T_{\rm C}.$$

The BCS theory predicts temperature variations of the energy gap near $T_{\rm C}$:

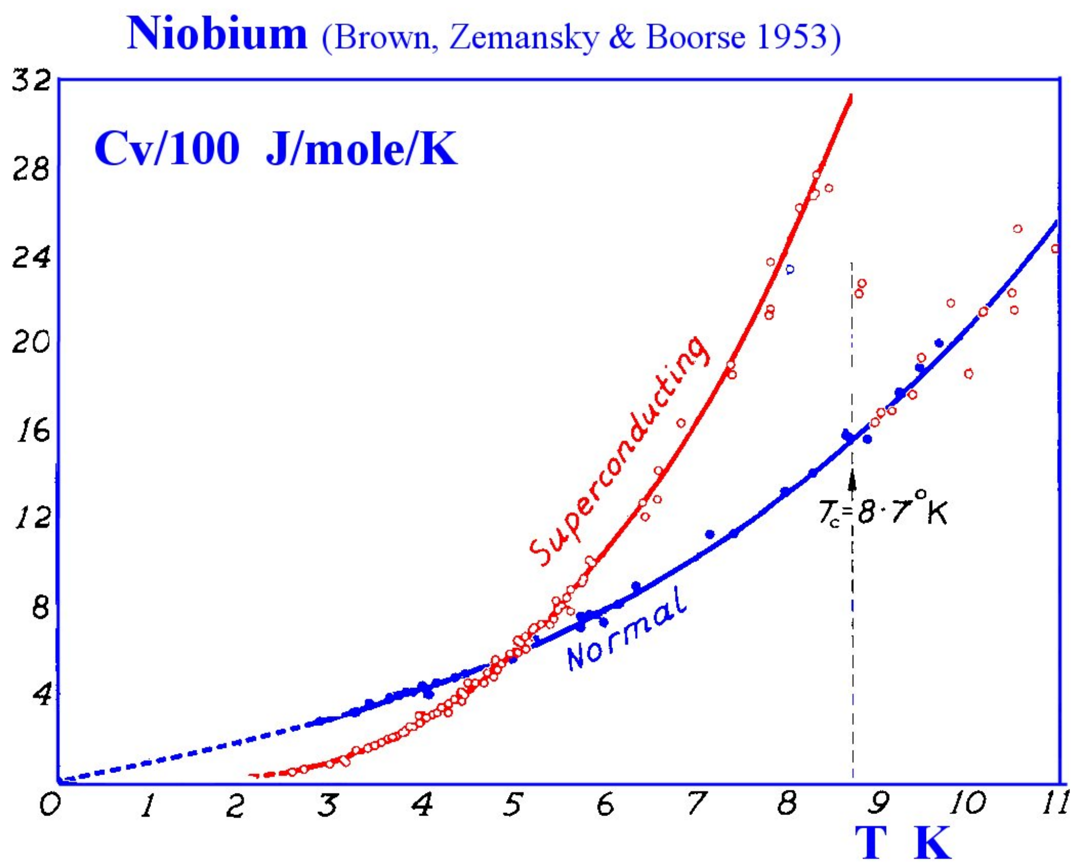
$$\frac{\Delta(T)}{\Delta(0)} = 1.74\left(1 - \frac{T}{T_{\rm C}}\right)^{1/2}$$

and the critical field

$$\frac{\mathcal{H}_{\rm C}(T)}{\mathcal{H}_{\rm C}(0)} = 1 - \left(\frac{T}{T_{\rm C}}\right)^2.$$

11.6 Experimental evidence for energy gap

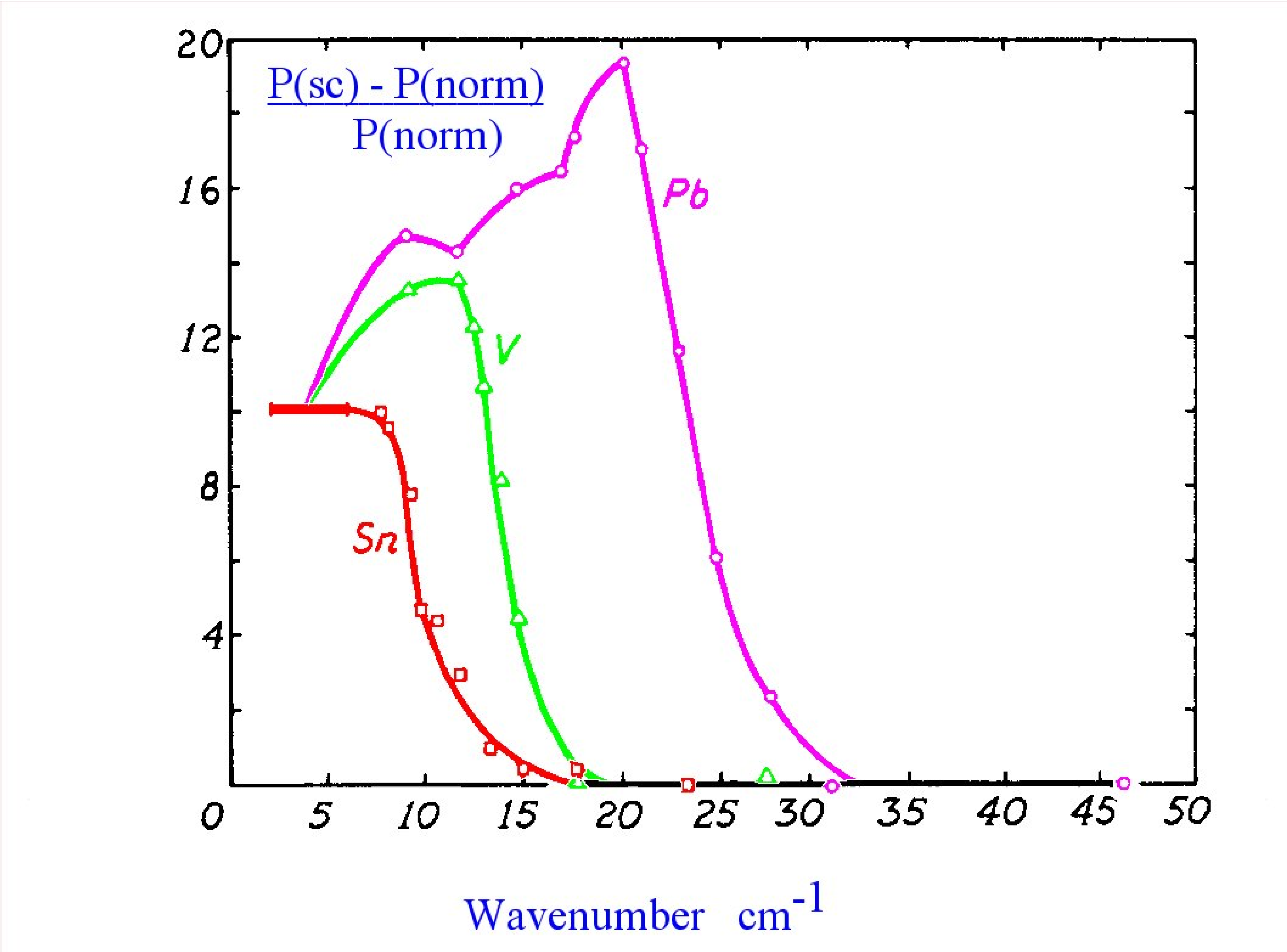
11.6.1 Specific heat

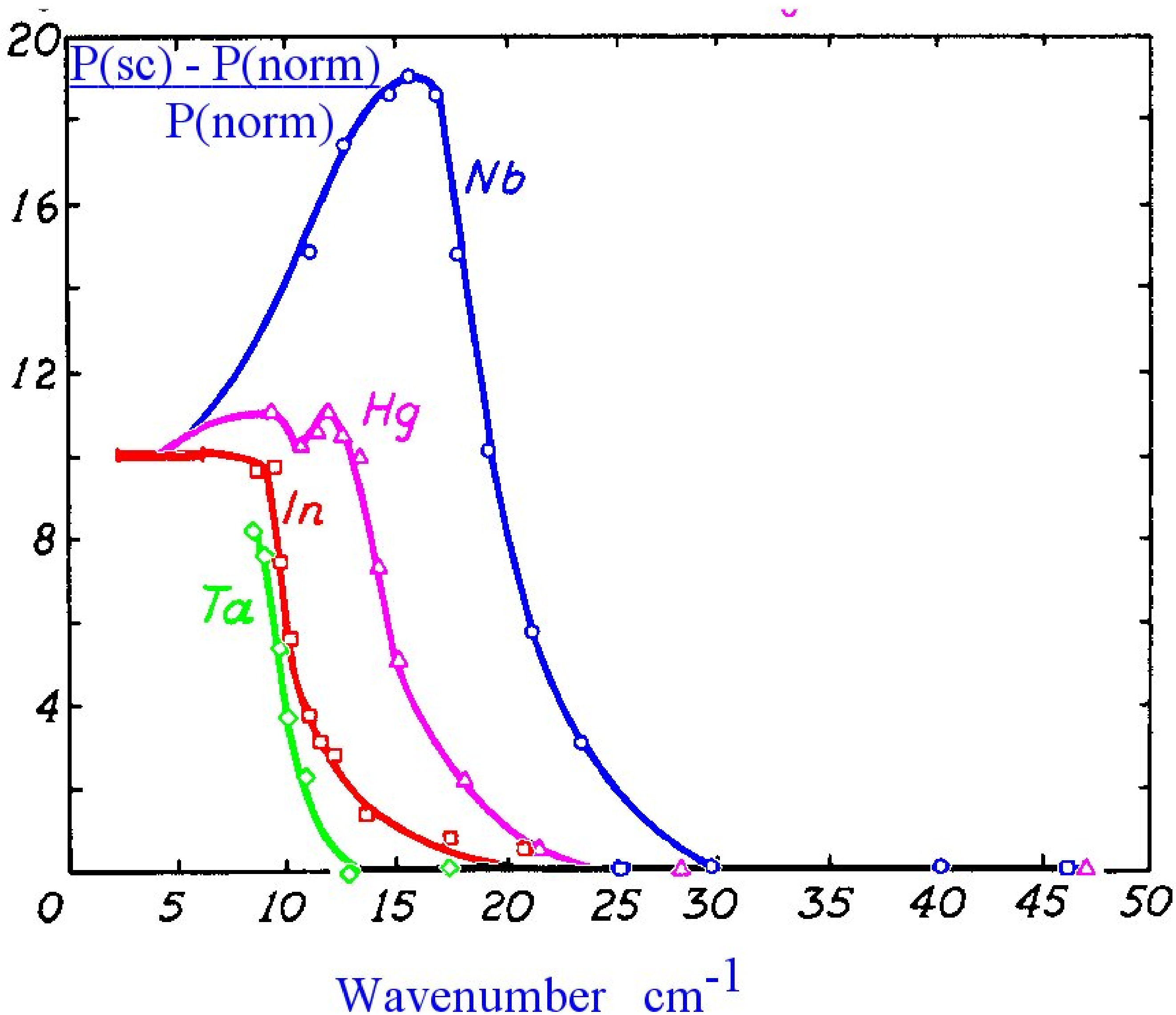


Number of electrons contributing to specific heat varies as

$$e^{-\Delta/(k_{\rm B}T)}.$$

11.6.2 Infrared absorption




P(sc) - P(norm)
P(norm)
Nb
Hg
In
Ta
20
16
12
8
4
0
5
10
15
20
25
30
35
40
45
50
Wavenumber cm-1

Values of energy gap deduced from infrared absorption (Richards and Tinkham 1960).

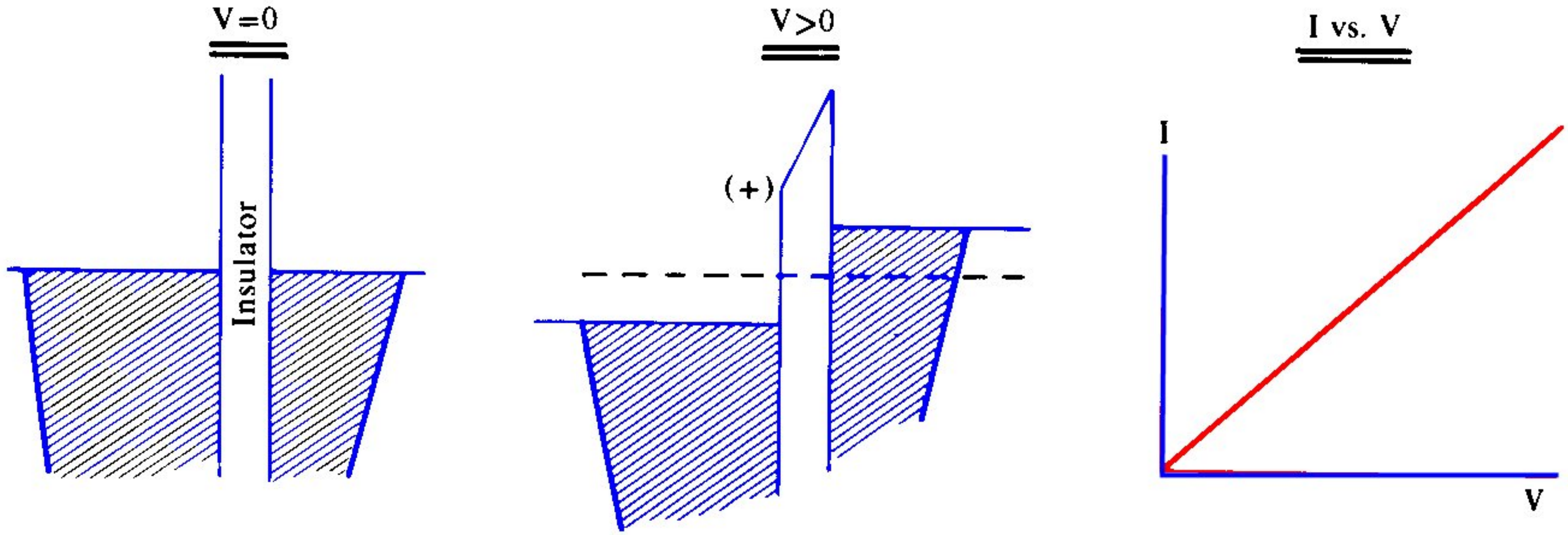
Metal	Threshold (cm^{-1})	T_C	$2\Delta/k_\mathrm{B}T_\mathrm{C}$
Ta	10	4.482	3.0
Nb	20	9.5	2.9
V	15	5.38	3.8
Pb	25	7.193	4.7
Sn	10	3.722	3.7
Hg	15	4.153	4.9
In	11	3.404	4.4

These results are in reasonable agreement with $2\Delta = 3.52 k_\mathrm{B} T_\mathrm{C}$.

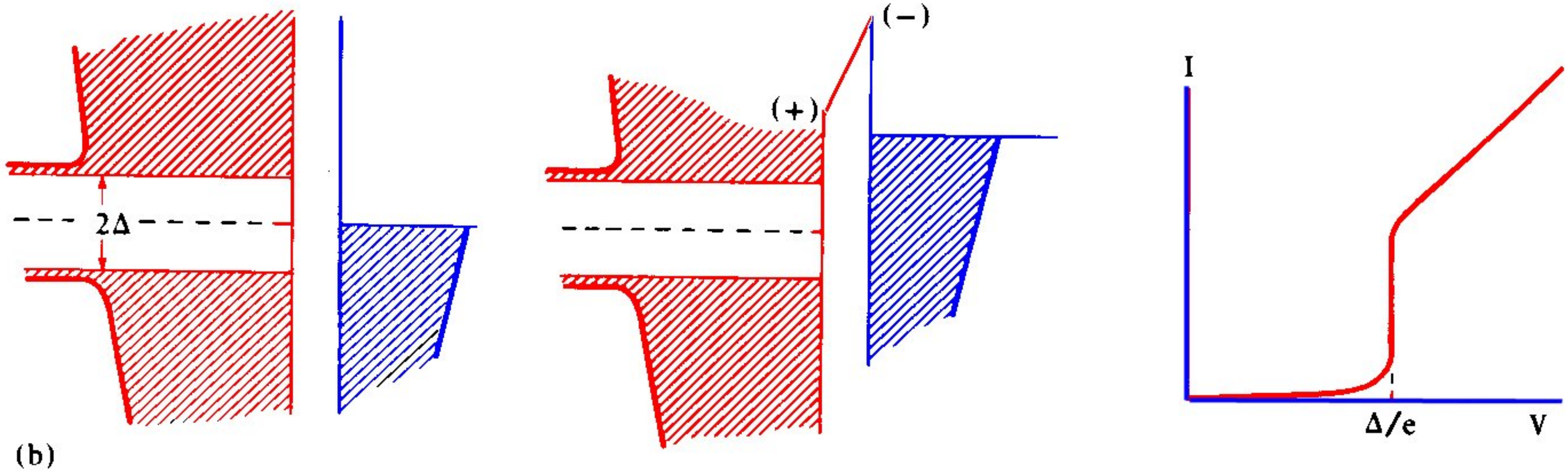
11.7 Tunnelling currents

Put two materials together with a very thin insulating layer between (often just an oxide layer) through which normal electrons can tunnel.

Two normal metals - linear $I - V$ relation.

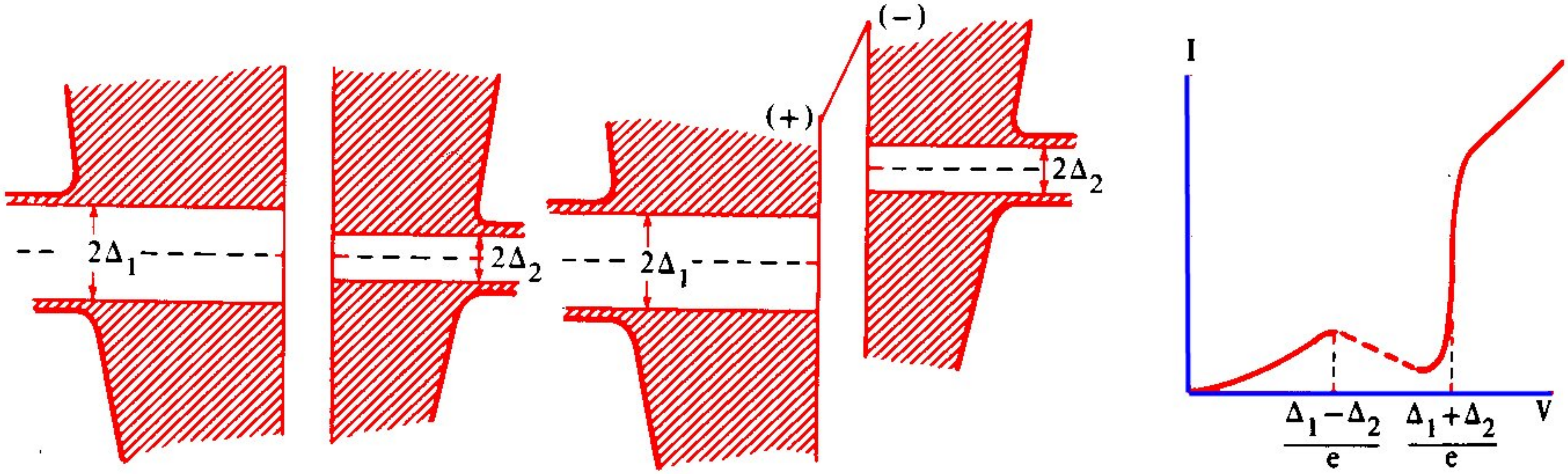


Superconductor-normal.

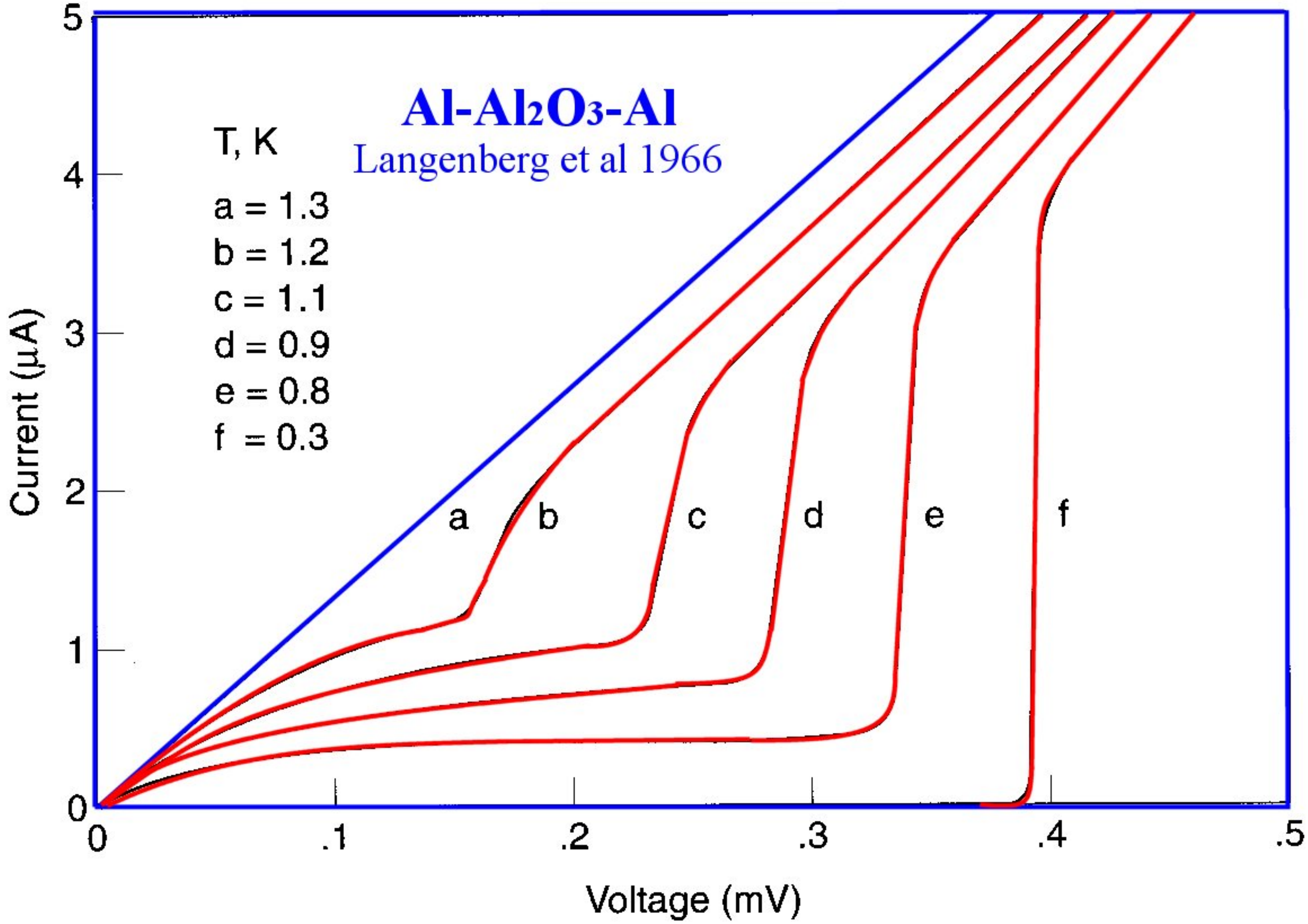


With no bias, there are no empty states to which electrons in the normal metal can pass.

Superconductor-superconductor



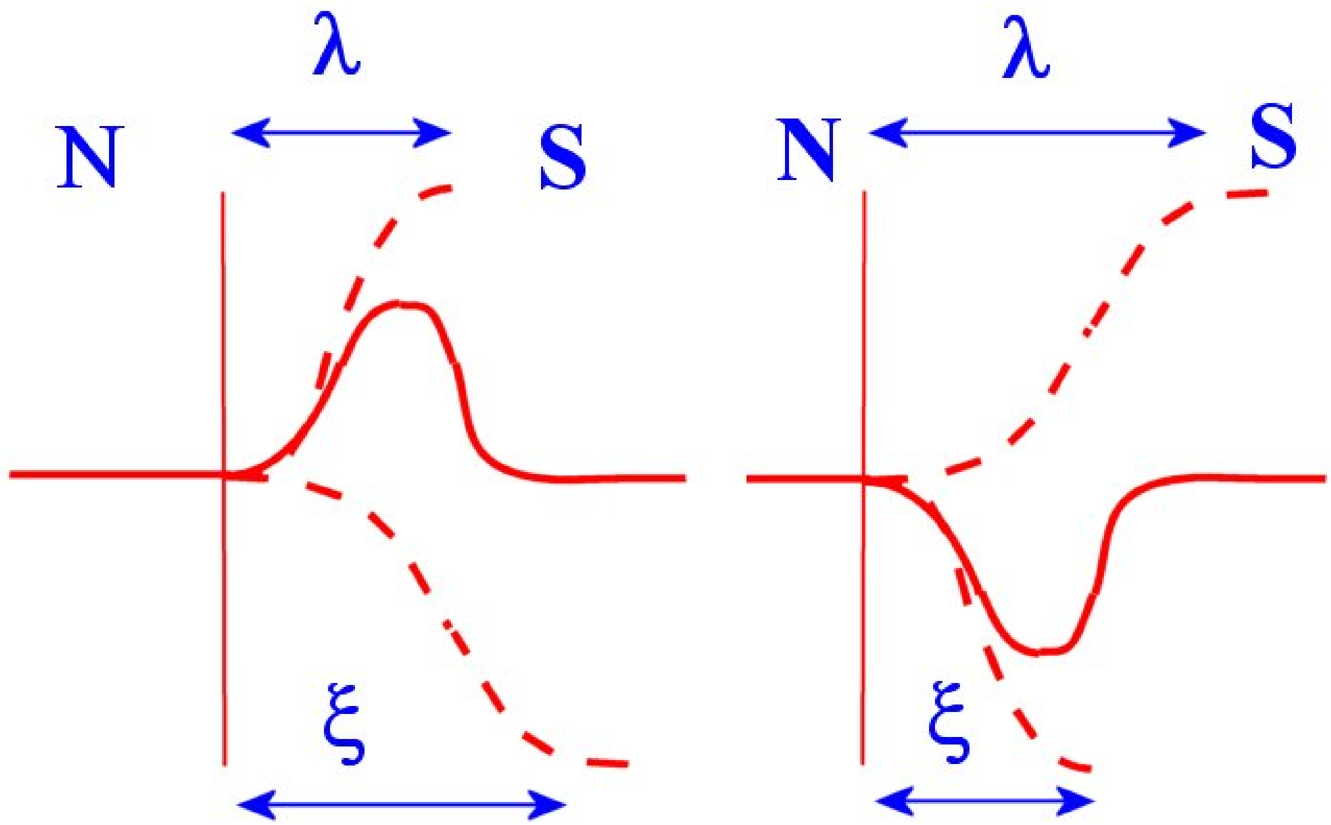
Small initial current from small number of excited electrons in material with smaller gap.



The threshold voltages allow us to measure △.

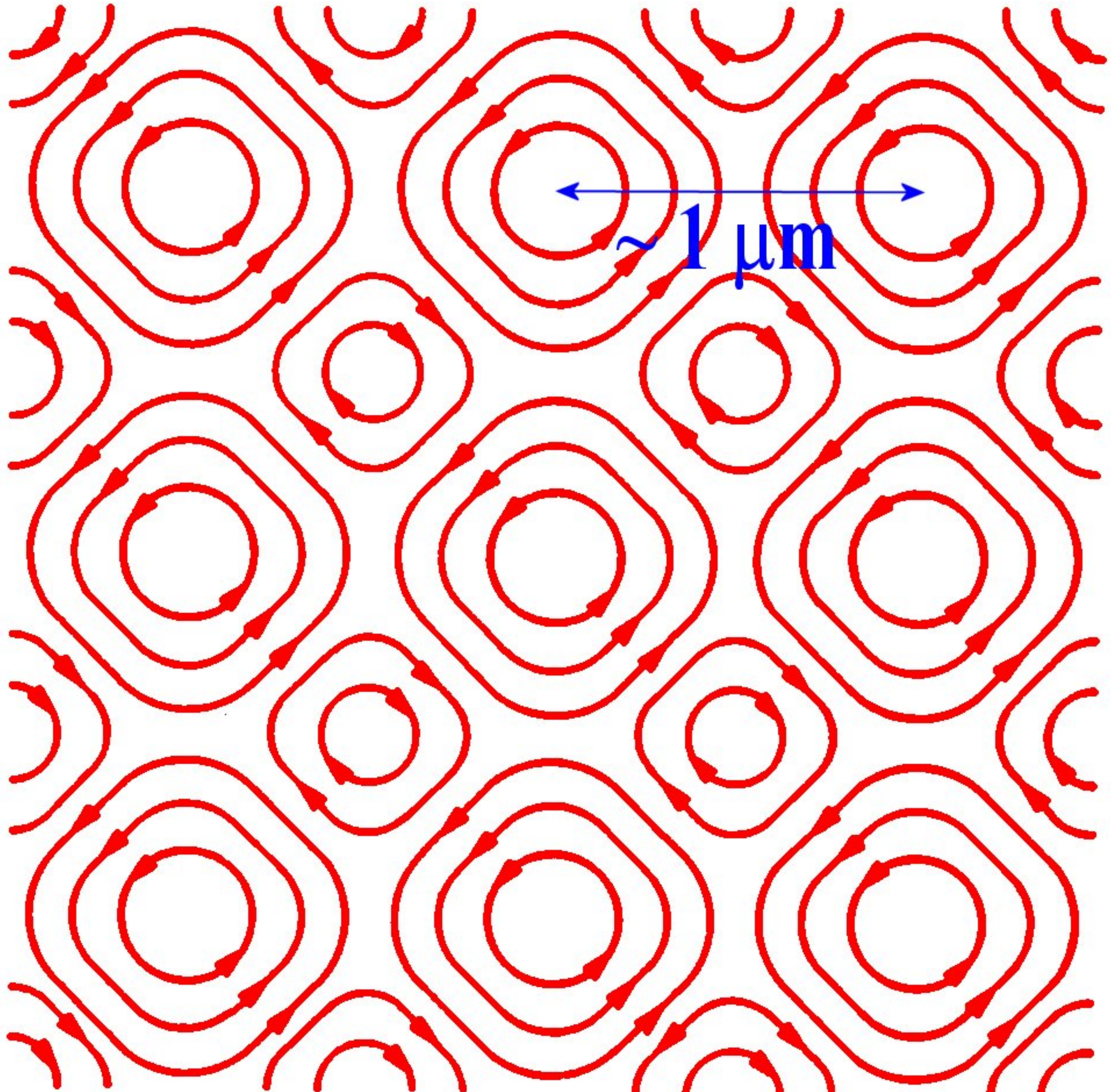
11.7.1 Type I and type II behaviour

When we apply a field, two effects compete: pairing reduces the free energy, whilst field penetration increases it. Each effect has a characteristic length scale, λ for flux penetration and ξ for pairing. At a phase boundary:



Type I: $\lambda < \xi$ **gives positive surface energy**
Type II : $\lambda > \xi$ **gives negative surface energy**

In Type II material lines of flux can penetrate one by one:



At the centre of each vortex of current is a normal region containing one quantum of magnetic flux, $h/(2e)$**.**

Vortex lines in $Pb_{0.98}In_{0.02}$ film in a magnetic field.