POTENTIALS IN PURE QCD ON 32^4 LATTICES

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A range of properties of pure SU(3) gauge theory are studied numerically on euclidean lattices of 32^4 points at β values of 6.29, 6.585 and 6.88. Potentials between static pairs of colour charges in the triplet, octet and sextet representations are calculated. The lowest energy (the A_{1g}) gluonic modes are considered for each of these plus the E_u mode for the triplet case. In the calculations, the use of extended operators on the lattice is examined. The results are assessed for scaling but do not agree adequately with asymptotic behaviour based on the two-loop β-function.

One of the most appealing formultions of quantum field theory is the lattice approach [1]. However, to justify rigorously any investigation based upon a discrete spacetime, it is necessary to consider a continuum limit. For pure QCD on the lattice, this can be achieved by taking the coupling β to infinity. Perturbative evaluations of the β-function dictate the behaviour to be expected of lattice quantities in this limit. Calculations have been made (for a review see ref. [2]) which give a impression of the required behaviour for β above about 6.3, but to check the reliability of this it is vital to continue to study couplings in this range. However, for larger and larger β the lattices considered must contain an ever increasing number of spacetime points. This ensures that the standard periodic boundary conditions used do not produce sizeable errors as the physical extent of the lattice is reduced.

This report gives the first results of a recent Monte Carlo study of pure QCD on the largest lattices yet constructed. Previously generated [3] configurations of 32^4 lattices at $\beta=6.29$, 6.585 and 6.88 have been examined in the light of recent advances [4] in the efficient construction of lattice operators which have high overlaps upon the required states. The lattices were equilibrated, using the Wilson action and a six-hit Metropolis algorithm, on Cyber 205 computers at the Universities of Manchester and Minnesota. For the greater part of the equilibration, the SU(3) matrices were stored using eight-parameter coding [5]; this was followed by a further 25 sweeps of each configuration with two-column matrix storage. Seven configuration at $\beta=6.29$ were selected on a basis of negligible correlation of the Polyakov line expectation value [6]: they all received a minimum of 1550 Monte Carlo sweeps and were separated by at least 200 sweeps. At $\beta=6.585$, eight configurations were selected, having a minimum equilibration of 500 sweeps and separation at least 200. The six configurations at $\beta=6.88$ were given at least 750 preliminary sweeps and were separated by 250.

The spin-averaged potentials between two static colour charges were evaluated for the triplet, octet and sextet representations [#1], using methods based on the well-known Wilson loop technique. In addition, the

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[#1] Further detail on these calculations is given in ref. [7].

triplet e_u hybrid potential was calculated. The spin dependent parts of these potentials have also been calculated, and will be reported elsewhere [8].

Apart from the problems of critical slowing down and the magnitude of the required computational effort, which both obstruct progress towards the taking of the continuum limit in QCD, there is the problem of achieving an adequate overlap between the required lattice state and the operator whose correlations are used to determine the state energy. For instance, the correlations of a euclidean lattice operator A at separation t can be analysed thus:

$$\langle A(t)A(0)\rangle = \frac{\sum_i \langle 0|A|i\rangle \langle i|A|0\rangle \exp(-E_i t)}{\sum_i \exp(-E_i t)} \to |\langle 0|A|\ell\rangle|^2 \exp[-(E_\ell - E_0)t] , \quad (1)$$

as $t\to\infty$, where E_i is the energy of state $|i\rangle$ and $|\ell\rangle$ is the lowest energy state which has overlap with $A|0\rangle$ ($|0\rangle$ is the vacuum). To reach the $t\to\infty$ limit it has proved useful [4,9,10] to optimise A such that the unwanted contributions are minimised. The limit is then achieved for smaller temporal separations of the operators where the correlations $\langle A(t)A(0)\rangle$ are larger and better determined. However, the process of taking a continuum limit corresponds to a contraction of the lattice structure of links and points relative to the physical length scales of the continuum spacetime it is supposed to represent. This implies an increasing spatial extension, in lattice spacings, of the physical wavefunctions. To achieve an adequate excitation of such a state as the coupling β is increased, it is necessary to consider operators A which are equally extended. The efficient construction of such extended state operators has been studied recently, and in both SU(2) and SU(3) on small lattices encouraging results have been reported [4]. We have sought to apply these ideas in a study which is closer to the continuum and where, therefore, the overlap problem is more severe. The method used is iterative and consists of the following: spatial links of a lattice configuration are replaced by constructions which have a greater physical extent but possess the group transformation properties of the original link. Such composite links have been called "blocked", "smeared", or "fuzzy" links [4,9]. The construction could involve, for instance, a lattice path (a product of links) which begins at one end of the link under

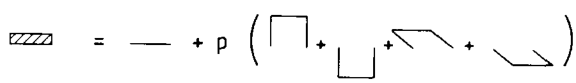

Fig. 1. Representation of the combination of spatial lattice paths involved in an iteration of the blocking scheme.

consideration and finishes at the other end. The blocking construction for the part of the study reported here used the combination of lattice paths shown in fig. 1, and written as

$$U^1_\mu(n) = U_\mu(n) + p \sum_{\pm\nu=1,3\neq\mu} U_\nu(n) U_\mu(n+a\hat{\nu}) U^\dagger_\nu(n+a\hat{\mu}) , \quad (2)$$

where $U_\mu(n)$ is a link emanating in direction μ from point n, $U^1_\mu(n)$ is the once blocked composite link and $U_{-\nu}(n) = U^\dagger_\nu(n-a\hat{\nu})$. Calculations were made for three choices of the constant p, from 0.5 to 1.5. Since it was found that the results for these three values of p differed by less than one per cent, the calculations reported below were all done with $p=0.5$.

The blocking procedure can be iterated to higher levels (U^1 being the first) to produce composite links involving contributions from a rapidly increasing number of basic paths. Such "links" have the required spatial extension and can be used to construct operators which are likely to have a high overlap with lattice states, for the reasons discussed earlier. Note, however, that U^1_μ in eq. (2) is not an SU(3) element.

To avoid numerical overflow of the blocking procedure, arising from the rapid increase in the number of paths contributing to the composite link with iterations of the blocking, the link at each iteration is normalised. Of several schemes considered, the best was the projection of the link upon the gauge group manifold. Some of our results, however, were obtained using a normalisation consisting of the condition

$$\mathrm{tr}\, U^B U^{B\dagger} = 1 , \quad (3)$$

where U^B is the composite link. This choice gave rise to overlaps almost as high as the SU(3) projection.

To calculate the potential between two colour sources at distance R apart, the expectation values of Wilson loops are evaluated from the available configurations at each β, using the blocked spatial links and unaltered temporal links. Such loops can be viewed as correlation functions, to lowest order in the fermion mass, of operators which model an extended gauge field structure terminating on the colour sources. The additional effect upon the results of using a variational basis [10] of paths of blocked spatial links was investigated and some improvement in overlap was achieved. However, the most important factor in the scheme was the number of iterations of

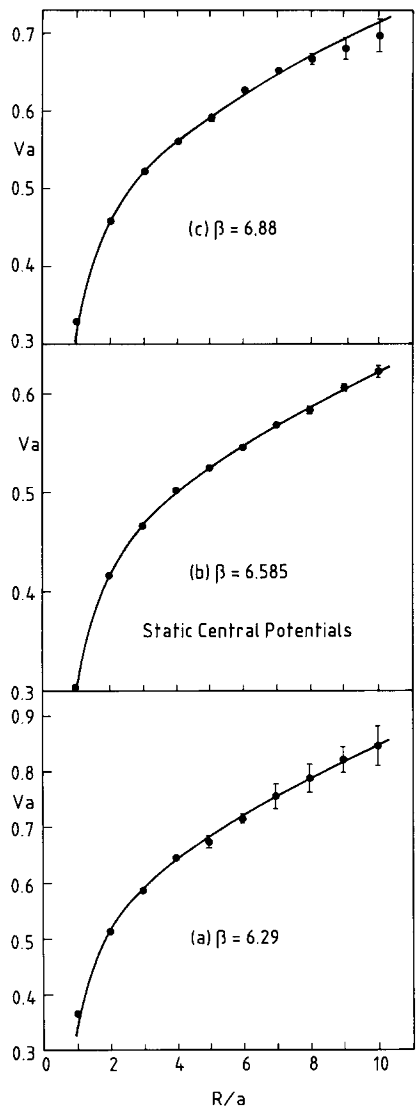

Fig. 2. The A_{1g} potential between triplet colour sources at (a) $\beta=6.29$, (b) $\beta=6.585$, and (c) $\beta=6.88$, calculated using the correlations of operators constructed at (a), (b) the second blocking level, and (c) the third blocking level.

blocking used, which heavily influences the degree to which the operator linking the colour sources extends in space.

Wilson loops W_i were required for each representation i of the SU(3) colour group considered. For the octet and sextet representations, the required loops were generated from the product L of blocked triplet links around the contour,

$$W_8 = |\mathrm{tr}\, L|^2 - \tfrac{1}{3}\mathrm{tr}\, LL^\dagger\,,$$

$$W_6 = \tfrac{1}{2}[(\mathrm{tr}\, L)^2 + \mathrm{tr}(L^2)]\,,$$

$$W_3 = \tfrac{1}{3}\mathrm{tr}\, L\,. \tag{4}$$

Note that L generally does not have the group multiplication properties of an SU(3) element; hence the explicit use of $LL^\dagger$ in eq. (4), for example. This procedure is far less time consuming than individually

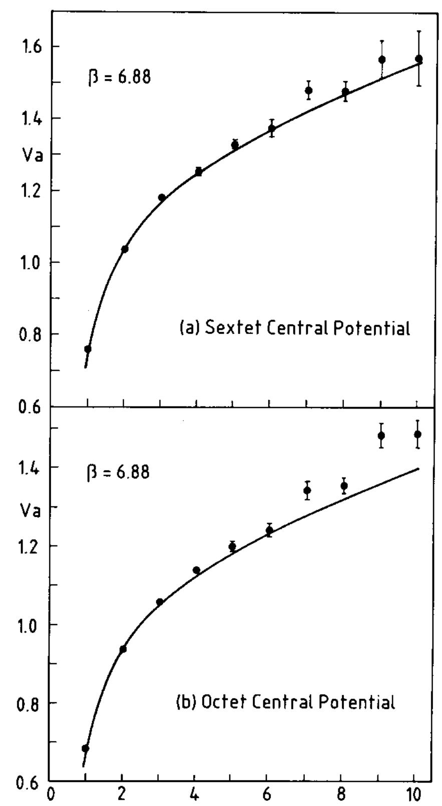

Fig. 3. The A_{1g} potential between (a) sextet and (b) octet colour source and sink at $\beta=6.88$, calculated using three levels of blocking.

Fig. 4. The combination of lattice paths used as an operator for the E_u gluonic mode.

converting each link in the product L to the desired representation and then constructing loops.

The correlation functions were sufficiently dominated by the first term in the expansion in eq. (1) for a temporal separation T of four lattice units. The triplet potentials V extracted at the three values of β are shown in figs. 2, 3 and 4. The fits are to the standard expression

$$V(R)=\alpha/R+KR+C\,, \tag{5}$$

and the dimensionless string tensions Ka^2, where a is the lattice spacing, are given in table 1. Assuming that a is given by the two-loop perturbative β-function, $\sqrt{K}$ can be tabulated in units of Λ_L, the lattice scale parameter. The failure to scale asymptotically is clear from the table. Such conclusion was reached previously [3] from these data, using planar Wilson loop methods and an assumed functional form for the triplet potential based on string models. The string tensions presented here are likely to be more reliable, however, since efforts have been made to ensure that the temporal behaviour of the Wilson loops is close to exponential at small T. Overlaps for $\beta=6.88$ are shown in table 2 for the optimised operators compared with those for straight line products of links joining the triplet sources.

Table 2
Overlaps O_e obtained using extended operators for the triplet A_{1g} potential at $\beta=6.88$, compared with the overlaps O_s obtained using a simple straight lattice path operator.

R/a	O_e	O_s
2	0.97	0.64
3	0.93	0.44
4	0.90	0.30
5	0.85	0.19
6	0.81	0.13
7	0.77	0.09
8	0.72	0.05
9	0.68	0.04
10	0.64	0.03

The higher representation colour potentials were found at each value of β to be consistent with a scaled-up triplet potential. Figs. 3a, 3b show the sextet and octet potentials, respectively, at $\beta=6.88$. The curves shown in fig. 3 represent the $\beta=6.88$ triplet potential in fig. 4, multiplied by a ratio of relevant Casimir operators. This factor is 2.25 for the octet and 2.5 for the sextet. The agreement supports the claims made elsewhere [11,12] that string tensions are proportional to the Casimir operator of the relevant colour representation. Of course, in both the octet and sextet cases, the potential is totally or partially screened at large enough distance, but the results given here are evidence that characteristically octet and sextet string tensions exist for at least some spatial separations of the colour charges. In addition, this behaviour is seen to persist at a β value very large by current standards. The results are too greatly influenced by statistical errors, however, to attempt to verify asymptotically scaling: it can only be concluded that it is no worse for these representations than it is for the triplet.

The E_u potential [13] is the last result to be reported here. It is of relevance to the spectrum of hybrid mesons and can be calculated through the correlations of the combination of path operators shown in fig. 4. The operators shown can be easily

Table 1
Triplet string tensions K at each β in dimensionless units and in units of Λ_L.

	A	Ca	Ka^2	$\sqrt{K}/\Lambda_L$
6.29	−0.292(34)	0.610(20)	0.0271(33)	97(6)
6.585	−0.265(14)	0.550(10)	0.0192(12)	114(4)
6.88	−0.214(13)	0.490(10)	0.0150(10)	141(5)

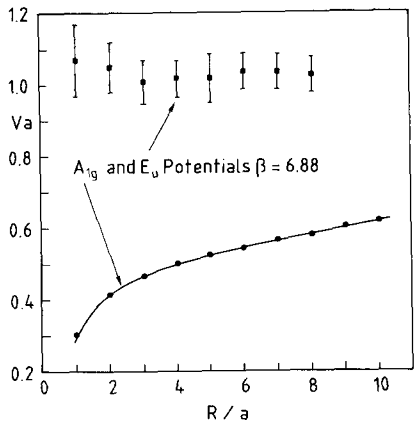

Fig. 5. The triplet E_u potential at $\beta=6.88$ compared to the A_{1g} potential. Operators were constructed at the third blocking level.

constructed from composite links and the consequent improvement in overlap studied. Fig. 5 is a plot of the E_u potential at $\beta=6.88$ compared with that of the A_{1g} state considered in fig. 2. The errors are sizeable and so an assessment of scaling with β is not very useful. The finding of a wide, flat potential is in accordance with that in ref. [13], but at a much higher β. The position of the plateau found in ref. [13] at $\beta=6.0$ would lie at about 0.8 on the vertical scale in fig. 5.

In conclusion, the results reported here confirm two things. Firstly, that operator smearing techniques can strongly influence the overlaps achieved upon the desired lattice states, and secondly that many features of pure QCD seen on lattices at around $\beta=6.0$ persist to β as high as 6.88. Asymptotic scaling, however, remains elusive.

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