# *Against simplification:*

## *Free choice with anaphora*

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# *Introduction*

# Roadmap

- Background: anaphora in disjunctive sentences.
- The problem: free choice with anaphora.
- Setting up the analysis: Bilateral Update Semantics and Partee disjunctions.
- Integrating Bilateral Update Semantics with Goldstein's (2019) semantic account of free choice.

# *Anaphora in disjunctive sentences*

Discourse anaphora is possible across disjuncts $\phi \vee \psi$ if $\phi$ contains an existential statement, and $\neg\phi$ contextually entails a witness to the existential (Barbara Partee).

(1) Either there isn't a$^{x}$ bathroom in this house,
or it$_{x}$'s in a funny place. $\neg\exists_x B(x) \vee F(x)$

You might be skeptical that (1) is a *bona fide* case of discourse anaphora. Two relevant observations:

- Formal Link Condition.
- Uniqueness inference (or lack thereof).

# Formal link condition

This is *bona fide* discourse anaphora — Partee disjunctions are subject to the *formal link condition*.

(2) a. ? Rob is married, and he'll bring them$_x$.

b. Rob has a$^x$ spouse, and he'll bring them$_x$.

(3) a. ? Either Rob isn't married or he'll bring them$_x$.

b. Either Rob *doesn't* have a$^x$ spouse
or he'll bring them$_x$.

Contrast between (2a) and (2b) parallels contrast between (3a) and (3b).

Like *bona fide* discourse anaphora, anaphora in Partee disjunctions doesn't give rise to an (obligatory) uniqueness inference — we can see this by adapting Heim's famous Sage plant sentence (Mandelkern & Rothschild 2020).

(4) a. Sue bought a$^x$ Sage plant,
and she bought 8 others along with it$_x$.

b. Either Sue didn't buy a$^x$ Sage plant,
or she bought 8 others along with it$_x$.

Mandelkern & Rothschild emphasize that this is incompatible with sophisticated E-type accounts of anaphora Partee disjunctions (i.e., **it**=[**the Sage plant**]; Heim 1990, Elbourne 2005). This will be important!

# Theoretical status of Partee disjunctions

Classical dynamic theories of discourse anaphora (e.g., Heim 1982, Groenendijk & Stokhof 1991) are unable to account for Partee disjunctions due to their treatment of negation.

Without going into detail, the crux of the problem: *Rob isn't married* and *Rob doesn't have a spouse* end up being semantically equivalent.

Several alternative frameworks have subsequently emerged which resolve this issue, although the details vary; see especially Krahmer & Muskens 1995, Gotham 2019, Elliott 2020, Hofmann 2019, 2022, Mandelkern 2022.

# *Free choice with anaphora*

The problem of Free Choice (FC) — how to validate (5) (Kamp 1973).

(5) **FC:** $\Diamond(\phi \vee \psi) \Rightarrow \Diamond\phi \wedge \Diamond\psi$

(6) You may have coffee or tea.

a. $\Rightarrow$ *You may have coffee and you may have tea*

Puzzle extends to other modal operators with existential force (Fox 2007).

Accounts of FC can be split into two main camps:

- Exhaustification accounts — FC is an implicature (Kratzer & Shimoyama 2002, Alonso-Ovalle 2005, Fox 2007, Bar-Lev & Fox 2017, Bar-Lev 2018, del Pinal, Bassi & Sauerland 2022).
- Semantic accounts — FC is a semantic entailment (Zimmermann 2000, Aloni 2022, Simons 2005, Willer 2018, 2019, Goldstein 2019, 2020).

Currently an open question which is the right approach to this phenomenon. Our talk will bear on this question, in favour of semantic accounts.

# Simplification in exhaustification accounts

What is relevant for our purposes is that all exhaustification accounts we're aware of involve reasoning over *simpler* 'domain' alternatives (Fox & Katzir 2011).

$$\phi, \psi \in \mathbf{ALT}(\phi \vee \psi)$$

Domain alternatives play an important role elsewhere, such as in the literature on ignorance inferences (Sauerland 2004) and distributive inferences (Crnič, Chemla & Fox 2015, Bar-Lev & Fox 2017).

# Simplification in semantic accounts

Semantic accounts are clearly not wedded to simplification, unlike exhaustification accounts, although many nevertheless place conditions on individual disjuncts.

Later on, we'll show how a concrete semantic account of FC can be tweaked in order to avoid the problems that arise due to simplification.

The problem of anaphora with free choice involves a Partee disjunction embedded under an existential modal.

(7) $\Diamond(\neg\exists_x P(x) \vee Q(x))$

As expected, sentences like (7) give rise to the FC inference (8a), corresponding to the first disjunct. Surprisingly however, they also give rise to the inference in (8b), whereas a simplification account can only generate (8c).

(8) a. $\Rightarrow \Diamond\neg\exists_x P(x)$

b. $\Rightarrow \Diamond\exists_x P(x) \wedge Q(x)$

c. $\not\Rightarrow \Diamond Q(x)$

# The problem, concretely i

First, an example with a possibility modal:

(9) It's possible that Tony doesn't have a$^x$ stash,
or that he hid it$_x$.

a. $\Rightarrow$ *It's possible that Tony doesn't have a stash.*

b. $\Rightarrow$ *It's possible that Tony has a stash and hid it.*

The problem extends to more classical cases involving deontic modals:

(10) You're allowed to (either) write no$^x$ squib, or submit it$_x$ before the final class.

a. ⇒ *You're allowed to write no squib.*

b. ⇒ *You're allowed to write a squib and submit it before the final class.*

# Sharpening the problem for simplification

One way of understanding what goes wrong — once we have a discourse-anaphoric dependency between the two disjuncts, the latter disjunct is no longer a **truthmaker** of the disjunctive sentence.

Accounts based on simplification implicitly rely on the domain alternatives of disjunction each being truthmakers of the disjunctive sentence.

An E-type approach to anaphora would potentially help get the right descriptive content in the latter disjunct (with local accommodation), i.e.:

(11) You're allowed to write no squib,
or submit it[**=the squib**] before the final class.

As we already showed in our discussion of donkey anaphora, an E-type approach isn't tenable for Partee disjunctions in the first place.

# Local accommodation?

Another way out would be to say that the *anaphoric* presupposition of the free pronoun is somehow locally accommodated.

**but:** anaphoric presuppositions *can't* easily be locally accommodated, cf. Barbara Partee's famous marble example.

(12) a. $1^x$ out of these 10 marbles is still missing.
It$_x$'s under the couch.

b. I've found 9 out these 10 marbles.
# It$_x$'s under the couch.

Partee disjunctions — a crucial aspect of the puzzle — are already problematic for many existing theories of discourse anaphora (with some notable exceptions, such as Gotham 2019, Hofmann 2019, 2022).

We adopt Elliott's (2020) *Strong Kleene* account of Partee disjunctions, which has the virtue of drawing a tight connection between presupposition projection and anaphora in disjunctive sentences.

As a proof of concept, we integrate Goldstein's (2019) semantic, homogeneity-based theory of FC with Elliott's account of Partee disjunctions.

# *Bilateral Update Semantics*

Our initial goal will be to set up a simple account of Partee disjunctions by setting up an update semantics which validates *Double Negation Elimination*.

(13) $\neg\neg\phi \iff \phi$

This will be necessary in order to account for how the *negation* of a negative statement can introduce a discourse referent. There is independent evidence that this is necessary (see especially Gotham 2019 for discussion).

(14) There's no way that Matt doesn't own a$^x$ smart shirt. It$_x$'s in his closet!

We'll accomplish this by setting up a Bilateral Update Semantics (BUS), in which an expression $\phi$ is associated with both a *positive update* $.[\phi]^+$ and a *negative update* $.[\phi]^-$ — see Elliott (2022) for details of the full system.

*Updates* are functions from Heimian information states (sets of world-assignment pairs) to information states.

The positive update $c[\phi]^+$ often (but not always) corresponds to the effect of asserting $\phi$ against a Heimian file context set $c$.

(15) $s[\text{it}_x \text{ is upstairs}]^+ := \{ (w, g) \in s \mid g_x \text{ is upstairs}_w \}$

(16) $s[\text{it}_x \text{ is upstairs}]^- := \{ (w, g) \in s \mid g_x \text{ isn't upstairs}_w \}$

Atomic sentences are associated with a positive/negative update which picks out the possibilities in $s$ at which the sentence is true/false respectively.

We assume that assignments are partial, which means that $s[\phi]^{+,-}$ doesn't always partition $s$.

In order to capture Heimian familiarity, we assume that $c[\phi]^{+,-}$ must partition $c$ in order for $\phi$ to be assertable at $c$.

The positive update of an existential statement introduces a discourse referent, just like in ordinary update semantics.

(17) $s[\text{there is a}^x \text{ bathroom}]^+ :=$
$\{\,(w,h) \mid (w,g) \in s, g[x]h \wedge h_x \text{ bathroom}_w\,\}$

Crucially, the negative update of an existential statement simply picks out possibilities in $s$ at which there is no bathroom, without introducing any anaphoric information.

(18) $s[\text{there is a}^x \text{ bathroom}]^- :=$
$\{\,(w,g) \in s \mid \text{there is no bathroom in } w\,\}$

In order to validate DNE, we can simply adopt the following "flip-flop" entry for negation (common in a bilateral setting).

(19) $s[\text{not } \phi]^+ := s[\phi]^-$

(20) $s[\text{not } \phi]^- := s[\phi]^+$

It's obvious that this entry validates DNE, since $s[\neg\neg\phi]^+ = s[\neg\phi]^- = s[\phi]^+$, and $s[\neg\neg\phi]^- = s[\neg\phi]^+ = s[\phi]^-$.

This means that, e.g., $s[\text{there's no}^x \text{ bathroom}]^-$ will introduce a bathroom discourse referent. This will be crucial for our account of Partee disjunctions.

In BUS, we cash out the Strong Kleene truth table as a recipe for constructing positive/negative updates of complex expressions.

| $\phi \vee \psi$ | $\psi_+$ | $\psi_-$ | $\psi_?$ |
|---|---|---|---|
| $\phi_+$ | $+$ | $+$ | $+$ |
| $\phi_-$ | $+$ | $-$ | $?$ |
| $\phi_?$ | $+$ | $?$ | $?$ |

**Figure 1:** Strong Kleene disjunction

Each $+, -$ cell is interpreted as an instruction to perform a successive update. In order to get the result of the positive update of $s[\phi \vee \psi]^+$, we take the union of all of the successive updates represented by the $+$ cells.

In order to make sense of the ? cells — which correspond to the ‘unknown’ truth-value in Strong Kleene trivalent logic — we must define a derivative notion — the ‘unknown’ update.

(21) $s[\phi]^? = \{ i \in s \mid i \not< s[\phi]^{+,-} \}$

In the simplest case, the unknown update picks out the parts of $s$ which are neither in the positive, nor the negative update. To illustrate its utility, consider the unknown update of an open sentence:

(22) $s[\text{it}_x\text{'s upstairs}]^? := \{ (w, g) \in s \mid g_x \text{ is undefined} \}$

(N.b. we can think of our bridge principle as a requirement that $c[\phi]^?$ is empty.)

# Partee disjunctions — the negative case

Let's take a simple Partee disjunction, and start with computing the negative update.

(23) Either there's no$^x$ bathroom, or it$_x$'s upstairs.

(24) $$\begin{aligned} s[\neg\exists_x B(x) \vee U(x)]^- &= s[\neg\exists_x B(x)]^-[U(x)]^- \\ &= s[\exists_x B(x)]^+[U(x)]^- \end{aligned}$$

(25) $$= \left\{ (w,h) \;\middle|\; \begin{array}{l} (w,g) \in s, g[x]h \\ \wedge\, h_x \text{ is a non-upstairs bathroom in } w \end{array} \right\}$$

N.b. that in BUS, de Morgan's equivalences go through — $\neg(\phi \vee \psi) \iff \neg\phi \wedge \neg\psi$, so "it's not the case that there is no bathroom or it's upstairs" is equivalent to "There's a bathroom and it's not upstairs" (by DNE).

The positive update is somewhat more involved. By the Strong Kleene truth-table, we must compute the following:

(26) $$s[\phi \vee \psi]^+ := s[\phi]^+[\psi]^+ \cup s[\phi]^+[\psi]^- \cup s[\phi]^+[\psi]^? \cup s[\phi]^-[\psi]^+ \cup s[\phi]^?[\psi]^+$$

Roughly, the first line corresponds to dynamically verifying the disjunction by the truth of the first disjunct, and the second line corresponds to dynamically verifying the disjunction by the truth of the second disjunct.

We'll go through these cases one by one for our bathroom sentence.

Let's assume that the first disjunct is true — since the second disjunct introduces no anaphoric information, its contribution is trivial:

(27) $s$[there is no bathroom]$^{+}$[it's upstairs]$^{+,-,?}$ $=$
$s$[there is no bathroom]$^{+}$

(if $\phi$ is atomic, then $s[\phi]^{+,-,?} = s$)

What if the first disjunct is *false* — by DNE, that means it will introduce a DR, and the truth of the disjunction is dependent on the second disjunct being *true*.

(28) $s$[there is no bathroom]$^{-}$[it's upstairs]$^{+}$ =
$s$[there's a bathroom]$^{+}$[it's upstairs]$^{+}$

The $s[\phi]^{?}[\psi]^{+}$ case is irrelevant, since the first disjunct (an existential statement) is bivalent.

We've computed the positive update of a Partee disjunction:

(29) $s$[there's no$^x$ bathroom or it$_x$'s upstairs]$^+$

$$= s[\text{there's a}^x \text{ bathroom}]^- \cup s[\text{there's a}^x \text{ bathroom}]^+[\text{it}_x\text{'s upstairs}]^+$$

$$= \{\,(w,g) \in s \mid \text{there's no bathroom in } w\,\} \cup \{\,(w,h) \mid (w,g) \in s, g[x]h, h_x \text{ an upstairs bathroom}\,\}$$

Possibilities where no bathrooms exist are retained, and bathroom-upstairs possibilities are associated with a bathroom discourse referent.

Remember that the negative update associates bathroom-not-upstairs possibilities with a bathroom discourse referent. This covers all scenarios — $s[.]^?$ is empty!

We've already mentioned that de Morgan's equivalences hold in BUS. By virtue of this and DNE, the following all end up being equivalent:

(30) Either there's no$^x$ bathroom, or it$_x$'s upstairs.

(31) It's not the case that there's (both) a$^x$ bathroom and it$_x$'s not upstairs.

(32) If there's a$^x$ bathroom, then it$_x$'s upstairs.

Partee disjunctions have *existential* truth-conditions — our bathroom sentence is true if there is an upstairs bathroom, even if another bathroom is not upstairs (here we depart from, e.g., Krahmer & Muskens 1995, Gotham 2019).

# *Accounting for FR with anaphora*

Now that we have a concrete account of discourse-anaphoric dependencies in disjunctive sentences, we're one step closer to accounting for FR with anaphora.

Since we've already developed an update semantics in order to account for Partee disjunctions, an account of FR which exploits update semantics is a natural fit — enter Goldstein's (2019) account.

The key idea behind Goldstein's semantic account of FR is that a disjunctive sentence semantically entails that each disjunct is possible.

(33) **Modal disjunction:** $\phi \vee \psi \Rightarrow \Diamond\phi \wedge \Diamond\psi$

Goldstein sketches an implementation of this idea in a simple update-semantic setting, following e.g., Veltman 1996. Here we simply translate Goldstein's account to BUS, with an important adjustment.

In update semantics, it is standard to treat epistemic modals as *consistency tests* on information states.

This idea can be translated straightforwardly into BUS:

(34) $s[\Diamond\phi]^{+} = s$ if $s[\phi]^{+} \neq \varnothing$ else $\varnothing$

(35) $s[\Diamond\phi]^{-} = s$ if $s \prec s[\phi]^{-}$ else $\varnothing$

(N.b. in order to state the negative update of "might $\phi$" we make use of the notion of *subsistence* from Groenendijk, Stokhof & Veltman 1996 — ask us more about this in the question period.)

The final step will be to modify our semantics for disjunction — we'll simply state a new entry $\overline{\vee}$ in terms of our existing semantics for $\vee$.

(36) $s[\phi\overline{\vee}\psi]^+ := s[\phi \vee \psi]^+$
if $s[\phi]^+[\psi]^{+,-,?} \neq \varnothing$ and $s[\phi]^{-,?}[\psi]^+ \neq \varnothing$
else $\varnothing$

(37) $s[\phi\overline{\vee}\psi]^- := s[\phi \vee \psi]^-$

The intuition here is that $\phi \vee \psi$ can only be true if *both ways of dynamically verifying the disjunction* are contextually consistent; the negative update remains the same as before.

Let's see how this derives FR with anaphora in a concrete case.

The inferences we want to derive:

(38) $\Diamond(\neg\exists_x B(x) \vee U(x))$
$\Rightarrow \Diamond\neg\exists_x B(x)$
$\Rightarrow \Diamond(\exists_x B(x) \wedge U(x))$

Let's consider what constraints the disjunctive sentence places on the input state $s$ (in order to be true):

(39) $\{(w, g) \in s \mid \text{no bathroom in } w\} \neq \varnothing$

(40) $\left\{ (w, h) \,\middle|\, \begin{array}{l} (w, g) \in s, g[x]h, \\ h_x \text{an upstairs bathroom in } w \end{array} \right\} \neq \varnothing$

So for the bathroom disjunction to be true, there should be at least one *no bathroom* possibility, and at least one *bathroom upstairs* possibility.

Now, the epistemic modal $\Diamond$ demands that there are some possibilities in $s$ at which the bathroom disjunction is true.

Since the bathroom disjunction itself places a contingency requirement on the input state, this will only hold if:

- The *no bathroom* possibilities in $s$ are non-empty.
- The *bathroom upstairs* possibilities in $s$ are non-empty

This guarantees that, whenever $s$ is consistent with $\neg\exists_x B(x) \vee U(x)$, $s$ is consistent with both $\neg\exists_x B(x)$ and $\exists_x B(x) \wedge U(x)$. FR with anaphora is thereby derived as a semantic entailment.

# *Conclusion*

# Summing up

Crucial ingredients:

- A dynamic account of Partee disjunctions which can deliver existential truth conditions — we went with BUS, but other good candidates: Mandelkern 2022, Hofmann 2019, 2022.
- An account of FR which treats it as a semantic entailment — we went with Goldstein's (2019) implementation of modal disjunction as a proof of concept.

## Open issues

- Generalization to non-epistemic modals — see Goldstein 2019 for details on how to generalize the account of FR we assume to non-epistemics.
- Can Partee disjunctions be used to problematize simplification-based accounts of other inferences involving disjunction? (ignorance inferences, distributive inferences, etc.).
- Is there a way to reconcile the exhaustification account and FR with anaphora? The only possibility we can think of is to assume a syntactic representation of the local context into the latter disjunct, but this raises many questions. We leave this as an open challenge.

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# *appendix*

# Another way out for simplification?

Yet another possible way out would be to say that the alternatives to disjunction are *closed under negation*.

The reasoning for $\phi \vee \psi$ could then go as follows:

- $\phi$ and $\psi$ are domain alternatives.
- Since alternativehood is closed under negation, $\neg\phi$ is also an alternative.
- $\neg\phi \wedge \psi$ is an alternative, via scalar substitution and substitution of $\phi$.

This will help generate the right inference, assuming that there is an account of discourse anaphora that validates double-negation elimination.

**But:** closure under negation *subverts* the original motivation for structural simplification — namely, the symmetry problem. See Katzir 2008, Fox & Katzir 2011 for details.