INTRODUCTION TO LOGIC

Answer all questions.

All questions have equal value.

PART A. BASIC LOGICAL NOTIONS

1. When are two propositions logically equivalent?

When it is impossible for them to have different truth values.

2. If two propositions are logically false, can you conclude from this that they are logically equivalent? Can you conclude that they are not logically equivalent? Explain your answer.

You can conclude that they are logically equivalent. It is not possible for either of them to be true, so it is not possible for them to have different truth values.

3. Can a logically invalid argument have a logically inconsistent set of premises?

No. Every argument with a logically inconsistent set of premises is logically valid. Since it is impossible for all its premises to be true, it is impossible for its premises to be true and its conclusion false,

PART B. SYMBOLIZATION IN SL

Symbolise in SL using the following key:

J: John will go to the party

M: Mary will go to the party

A: Ann will go to the party

4. If John doesn't go to the party, then either Mary or Ann won't go.

$\sim J \supset (\sim M \vee \sim A)$

5. Mary won't go to the party unless Ann goes.

$\sim A \supset \sim M$, or $M \supset A$, or $\sim M \vee A$

6. Neither John nor Mary will go to the party.

$\sim(J \vee M)$

PART C. SYNTAX AND SEMANTICS OF SL

7. Show that the string of symbols $\sim(A \supset \sim B)$ is a sentence of SL.

By the base of the definition of sentence, (1) A is a sentence and (2) B is a sentence. By the negation inductive clause, it follows from (2) that ~B is a sentence. By the conditional clause, it follows from (1) and (3) that (4) $(A \supset \sim B)$ is a sentence. And again by the negation clause it follows that $\sim(A \supset \sim B)$ is a sentence.

8. If we symbolise a logically true proposition in SL, will its symbolization always be truth-functionally true? Justify your answer.

No. There are logically true propositions whose logical truth is not due to truth functional composition. The symbolization in SL of those propositions won't in general be truth functionally true.

9. Use the truth-table method to determine whether the argument

$A \supset (\sim B \vee C)$

(A ⊃ B) ⊃ C

is truth-functionally valid. Explain how you obtain your answer.

A	B	C	A	⊃	(~	B	∨	C)	(A	⊃	B)	⊃	C	
T	T	T	T	**T**	F	T	T	T	T	T	T	**T**	T	
T	T	F	T	**F**	F	T	F	F	T	T	T	**F**	F	
T	F	T	T	**T**	T	F	T	T	T	F	F	**T**	T	
T	F	F	T	**T**	T	F	T	F	T	F	F	**T**	F	
F	T	T	F	**T**	F	T	T	T	F	T	T	**T**	T	
F	T	F	F	**T**	F	T	F	F	F	T	T	**F**	F	
F	F	T	F	**T**	T	F	T	T	F	T	F	**T**	T	
F	F	F	F	**T**	T	F	T	F	F	T	F	**T**	F	

The argument is nor truth-functionally valid, as there is one truth value assignment making the premise true and the conclusion false.

PART D. SD DERIVATIONS

10. Derive in SD the conclusion ~B from the premise B ⊃ (A & ~B).

1| B ⊃ (A & ~B)
2| |B Assumption
3| |A&~B 1, 2 ⊃E
4| |B 2R
5| |~B 3&E
6|~B 2-5 ~I

11. Derive in SD the conclusion A ⊃ B from the premises (A & ~B) ⊃ (~B & C) and C ⊃ ~A.

1|(A & ~B) ⊃ (~B & C)
2| C ⊃ ~A
3| |A Assumption
4| | |~B Assumption
5| | | A & ~B 3, 4 &I
6| | |~B & C 1, 5 ⊃E
7| | |C 6 &E
8| | |A 3R
9| | |~A 2, 7 ⊃E
10| |B 4-9 ~E
11| A ⊃ B 3-10 ⊃I

12. Derive in SD the conclusion (A ⊃ ~B) ⊃ (B ⊃ (~A ∨ ~C)) from no premises.

1| | A ⊃ ~B Assumption
2| | |B Assumption
3| | | |A Assumption
4| | | |B 2R
5| | | |~B 1, 3 ⊃E
6| | |~A 3-5 ~I
7| | |~A ∨ ~C 6 ∨I
8| | B ⊃ (~A ∨ ~C) 2-7 ⊃I
9|(A ⊃ ~B) ⊃ (B ⊃ (~A ∨ ~C)) 1-8 ⊃I

PART E. SYMBOLIZATION IN PLE

Symbolize in PLE using the following key:

U.D.: People

Tx: x is tall

Lxy: x likes y

f(x): x's father

13. Someone likes every tall person.

$(\exists x)(\forall y)(Ty \supset Lxy)$

14. No tall person likes his/her father.

$\sim(\exists x)(Tx \,\&\, Lxf(x))$

15. There are at least two tall persons.

$(\exists x)(\exists y)((Tx \,\&\, Ty) \,\&\, \sim x = y)$

PART F. SYNTAX AND SEMANTICS OF PLE

16. Explain informally the universal quantifier clause of the definition of the truth value of a formula in an interpretation for a variable assignment.

The clause needs to specify how the truth value of $(\forall x)\alpha$ (i.e. the output of the inductive clause for $\forall$ of the definition of formula) in an interpretation I for a variable assignment s depends on the truth value of α (i.e. the input of the clause) in I for any variable assignment. It says that $(\forall x)\alpha$ is true in I for s just in case α is true in I for any variable interpretation that differs from s at most in what element of the universe (of I) it pairs with x (i.e. for s(a/x), for every a in the universe of I).

17. Find an interpretation in which the sentence $(\exists x)(\exists y)(Rxy \,\&\, \sim Rxf(y))$ is true and one in which it is false.

True: UD positive integers. Rxy x is greater than y. f(x) the successor of x.

False: UD positive integers. Rxy x is less than y. f(x) the successor of x.

18. If the symbolisation of a proposition into SL is truth-functionally indeterminate, does it follow that its symbolisation into PLE is quantificationally indeterminate? Explain your answer.

No. If p is a logically true or logically false proposition, whose logical truth or falsehood is due not to truth functional combination, but to aspects of its structure grasped by predicate logic, then its symbolization into SL will be truth functionally indeterminate but its symbolization into PLE will not be quantificationally indeterminate.

PART G. PD DERIVATIONS

19. Derive in PD the conclusion $(\forall x)((Ax \,\&\, Bx) \supset Cx)$ from the premise $(\forall x)(Ax \supset Cx)$.

1	$(\forall x)(Ax \supset Cx)$	
2		$Aa \,\&\, Ba$	Assumption
3		Aa	2 &E
4		$Aa \supset Ca$	1∀E⊃
5		Ca	3,4 ⊃E
6	$(Aa \,\&\, Ba) \supset Ca$	2-5 ⊃ I
7	$(\forall x)((Ax \,\&\, Bx) \supset Cx)$	6 ∀I

20. Derive in PD the conclusion ~(∀x) Ax from the premise (∃x) ~Ax.

1	|(∃x) ~Ax	
2	| |~Aa	Assumption
3	| | |(∀x) Ax	Assumption
4	| | |Aa	3 ∀E
5	| | |~Aa	2R
6	| |~(∀x) Ax	3-5 ~I
7	|~(∀x) Ax	1, 2-6 ∃E

21. Derive in PD the conclusion (∀x)(∃y)(Ay ⊃ Ax) from no premises.

1	| |Aa	Assumption
2	| |Aa	1R
3	|Aa ⊃ Aa	1-2 ⊃I
4	|(∃y)(Ay ⊃ Aa)	3∃I
5	|(∀x)(∃y)(Ay ⊃ Ax)	4∀I