# 1. PLATO'S PROBLEM

The mathematics of Plato's time (428 – 348 BC) was mostly geometry and arithmetic, and geometry was by no means the minor partner. Plato himself put great emphasis on geometry: he mentions it repeatedly in *The Republic* and his extended example of knowledge acquisition in *The Meno* is a geometrical example.

*'Popular' v. mathematical geometry*

Geometry began as a stock of techniques for solving problems. Popular geometry is practical knowledge with an *empirical* basis: techniques and rules get established if they are found to work *on the basis of observation and measurement*.

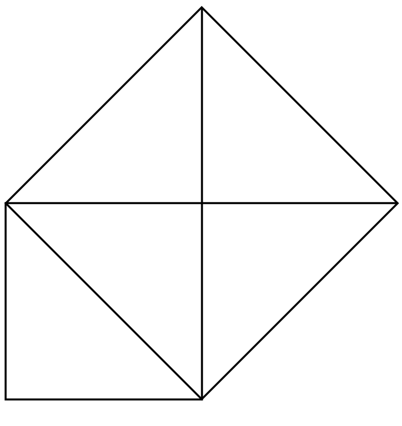
Plato held that there was another subject, related to popular geometry, that was *non*-empirical, the geometry of the mathematicians. Why? Because

(1) mathematicians did not arrive at geometrical theorems by observational checking and measurement,

(2) their theorems are not strictly true for perceptible objects.

## Discovery in mathematical geometry

(1) If geometrical truths are not discovered by observation and measurement, how are they discovered? Plato presents a justly famous example of geometrical discovery in the *Meno* (82b-85c). He shows how one might arrive at knowledge of the theorem that the area of the square on a diagonal of a square is twice as big.



This way of arriving at a geometrical truth does not involve measurement. Indeed it is not true for the actual figures in the diagram, so measuring would not help. Visual experience plays *some* role, but not the role of evidence. If visual experience were used as evidence, the evidence would not support the claim, for two reasons.

(i) A single instance is insufficient support for a universal claim.

(ii) We know that the diagram does not give us even one instance, because it is geometrically imperfect — edges bumpy and fuzzy, vertices formed from endpoints that do not exactly coincide etc.

In Plato's view, the role of the visual experience is to *prompt the mind* bringing certain truths to our awareness; from those truths we reach new geometrical knowledge by deductive reasoning.

**Geometry and the perceptible world**

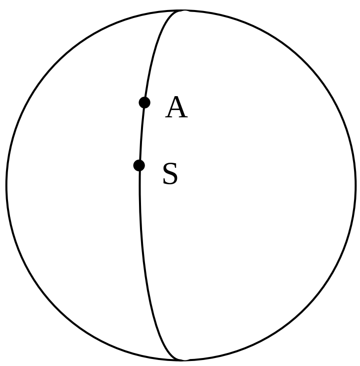
What about the second claim, that

(2) geometrical theorems are only approximately true of perceptible objects ?

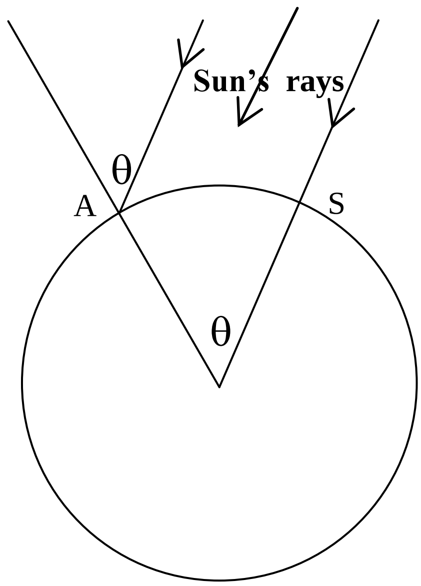
Here is an example of the application of geometry in ancient times.

*Determining the circumference of the earth*

The ancient Greeks already knew that the earth was not flat. They believed that it was roughly spherical. Eratosthenes, the polymath librarian of Alexandria, sought to determine the circumference of the earth.



For simplicity Eratosthenes assumed that the earth is perfectly spherical and that rays from the sun reaching the earth simultaneously were perfectly parallel, assumptions he thought would produce no significant error. The Nile runs through a twon then known as Syene to Alexandria. Eratosthenes knew that the Nile runs roghly North-South, so that part of the Nile between Syene and Alexandria is part of a great circle, i.e. a circumference of the earth, passing through the poles.




The distance along the arc between Alexandria and Syene was known (approx 5000 stadia). Now consider the angle $\theta$ at the earth's centre subtended by this arc. Clearly, the length of that arc AS is to the earth's circumference C as angle $\theta$ is to 360°:

$AS : C :: \theta : 360$

$AS / C = \theta / 360.$

As the distance AS was known, the task was to find angle $\theta$.

Here is how Eratosthenes found it. He knew that Syene had a very deep well whose water is touched by sunlight only at noon on the longest day of the year, so that at that point the sun would be directly over Syene. He realised that the angle $\theta$ subtended at the centre of the earth by the arc AS would be the same as the angle of the sun's inclination to the vertical in Alexandria when it was directly over Syene (as on OH 3). So at noon one midsummer day he measured the sun's inclination to the vertical in Alexandria, and found it to be 7°12'. There are 60' (minutes) in a degree, so that is one fifth or 0.2 of a degree. So the sun's inclination to the vertical in Alexandria at noon on midsummer's day i.e. the angle $\theta$ is 7.2°. Now we can feed this into our equation

$$\frac{AS}{C} = \frac{\theta}{360}$$

$$\frac{5000}{C} = \frac{7.2}{360}$$

By simple arithmetic we get

$$\frac{5000}{C} = \frac{1}{50}$$

So the circumference of the earth is 50 times the length of the arc AS:

$$\therefore C = 50 \times 5000 = 250,000 \text{ stadia}$$

To get this result, Eratosthenes makes some <u>simplifying assumptions</u>:

- the earth is a perfect sphere;
- the sun's rays reaching the earth are perfectly parallel.

These assumptions are needed in order to apply the <u>relevant theorems</u>

- Arc / Circumference = Angle subtended / 360°;
- When a line crosses a pair of parallel lines, corresponding angles are equal.

Those theorems are only *approximately* true of the physical situation, as the earth does not have a perfectly circular circumference and is not a perfect sphere, and the sun's rays reaching the earth on not always perfectly parallel. This illustrates the second claim:

> (2) The theorems of geometry are only approximately true for perceptible objects.

**Plato's Problem**

The same is true of other theorems, such as the geometrical theorem in the *Meno*.

> The area on a diagonal of a given square = 2 × the area of the given square.

They are only *approximately* true of perceptible things. This is because a perceptible square is only *approximately* flat, with only *approximately* straight borders, with *approximate* points as vertices, and so on. This was Plato's view. He held that perceptible objects do not really instantiate geometrical properties: nothing perceptible has a perfectly plane surface, or a perfectly straight edge; nothing perceptible is perfectly spherical or perfectly circular, not even planetary orbits (*Rep* VII 529c-530a; *VIIth Letter* 343a).

This leads directly to Plato's Problem. Geometry is a body of truths about real things. But perceptible things, to which geometry is applied, do not instantiate geometrical properties and relations, because of their imperfections. So geometry is not true of perceptible things. So what *is* geometry about? What are the lines, triangles, circles, spheres, etc. that geometry talks about? Plato's problem is, in short, *What are the objects of geometry?*

**Plato's response**

Plato, or at least his followers, postulated geometrical entities that were *intermediate* between perceptible things and Forms. A geometrical circle is perfectly circular unlike any perceptible circle; but as there is a plurality of geometrical circles (one for each point and distance pair) but only one Form of the circle, they cannot all be that Form.

This is unsatisfactory. A special metaphysical category is invented, the *intermediates*, simply in order to cater for geometric objects, on the grounds that it cannot be accommodated within the categories for which there is an independent basis. This is too *ad hoc* to command conviction.

**Where to from here?**

*What are the objects of geometry*? What other response are to this problem? Here are two we will consider later on:

– Geometry is not true of anything.

– Geometry is true but what it is true of are not objects.